On the generalization of Reissner plate theory to laminated plates

Arthur Lebée[(1)], Karam Sab[(1)]

(1) *Laboratoire Navier, École des Ponts ParisTech, IFSTTAR, CNRS, Université Paris-Est.*

École Nationale des Ponts et Chaussées, 6 et 8 avenue Blaise Pascal.
77455 Marne-la-Vallée cedex2

Keywords : Higher-order plate models, Reissner plate model

The classical theory of plates, known also as Kirchhoff-Love plate theory is based on the assumption that the normal to the mid-plane of the plate remains normal after transformation. This theory is also the first order of the asymptotic expansion with respect to the thickness [2]. Thus, it presents a good theoretical justification and was soundly extended to the case of periodic plates [1]. It enables to have a first-order estimate of the macroscopic deflection as well as local stress fields. In most applications the first-order deflection is accurate enough. However, this theory does not capture the local effect of shear forces on the micro-structure because shear forces are one higher-order derivative of the bending moment in equilibrium equations ($Q_\alpha = M_{\alpha\beta,\beta}$).

Because shear forces are part of the macroscopic equilibrium of the plate, their effect is also of great interest for engineers when designing structures. However, modeling properly the action of shear forces is still a controversial issue. Reissner [9] suggested a model for homogeneous plates based on a parabolic distribution of transverse shear stress through the thickness (Reissner-Mindlin theory). This model performs well for homogeneous plates and gives more natural boundary conditions than those of Kirchhoff-Love theory. Thus, it is appreciated by engineers and broadly used in applied mechanics. However, the direct extension of this model to laminated plates raised many difficulties as well from axiomatic derivations (*a priori* assumptions on local fields) as from asymptotic approaches.

Recently Lebée and Sab [3] suggested a new plate theory introducing a generalized shear force (the gradient of the bending moment) instead of the conventional shear force which is the divergence of the bending moment. This plate theory gives very good results with laminated plates [4] and was also extended to periodic plates [5, 6].

The derivation of this new plate theory was inspired from Reissner's ideas. However, it did not follow exactly the original derivation from Reissner. Whereas Reissner derived a strictly 3D statically compatible field and applied the principle of minimum of complementary energy, the stress distribution introduced in [3] satisfied the equilibrium equations only at a high order. In the present work [7, 8], an exactly statically compatible stress field is derived following the procedure from Reissner [9]. This requires the introduction of the first and second gradients of the bending moment. Applying the principle of minimum of complementary energy leads to a higher order plate theory with at most 15 kinematic degrees of freedom.

Even if this new plate theory called "Generalized Reissner" plate theory involves a rather large number of degrees of freedom, its derivation from the rigorous application of the principle of minimum of potential energy presents a number of noticeable advantages. First, the definition of generalized displacements as function of 3D displacement is clearly established. Second, it is possible to reconstruct a 3D displacement localization as function of plate variables which is consistent energetically. Third, the minimum of complementary energy ensures that this displacement field is actually an upper bound of the exact 3D displacement in the sense of external work. Fourth, when the plate is simply supported, it is illustrated empirically that the deflection of this new plate model converges faster with respect to the slenderness of the plate than the Kirchhoff-Love plate model. Finally, the Bending-Gradient theory may be fully recovered, locking the higher order kinematic degrees of freedom.

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