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MIGHT BOUNDARY HOMOGENIZATION HELP TO UNDERSTAND FRICTION?

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1. PRESENTATION OF THE RESULTS

The importance of the role played by the morphology of contacting surfaces in the frictional behaviour of deformable bodies has long been recognized (Green and Williamson [1], Tabor [2]). The roughness and the material properties of the body located in the vicinity of the contact surface are small scale characteristics which can tremendously influence the large scale behavior. The aim of the present study is to establish a contact law directly involving the surface state. Several models based upon a mechanical analysis of the deformation of asperities have been proposed in the literature. Some relevant, but non exhaustive, studies addressing this problem are [1] [2] [3] [4] and their references. A mathematical analysis of this problem based on boundary homogenization was presented by Sanchez and Suquet [5] in a setting restricted to the linear behaviour of contacting materials. The results were derived by means of formal asymptotic expansions, under the assumption that the material was linearly elastic. They are extended here to more general types of behaviour ranging from linear elasticity to deformation plasticity, defined in terms of a convex deformation energy j and their derivation is based on the rigorous use of Γ-convergence (convergence of functionals). The main aim of the present study is to present this improved mathematical tool for boundary homogenization in a deliberately simplified mechanical framework. Large strains and large displacements at small scale are therefore disregarded and no attention is paid here to the possible presence of a third body.

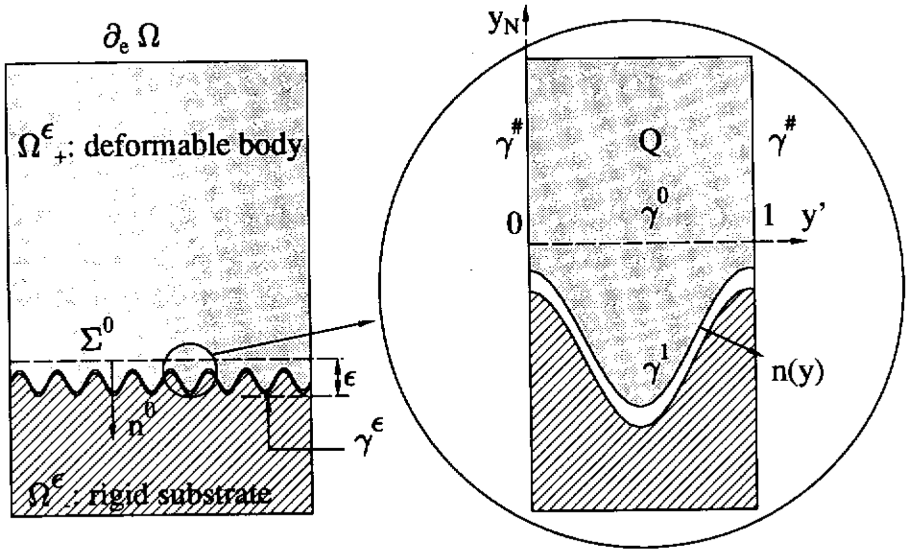


Fig. 1: Bodies in contact and unit cell
(For clarity the two surfaces in contact have been drawn separately. They are materially different although geometrically coincident)

A deformable body Ω_+^ϵ is in contact with a rigid substrate along an undulated boundary γ^ϵ (the notations will be detailed in section 2). The amplitude and the period of the asperities are proportional to a small parameter ϵ . The contact on γ^ϵ is controlled by the frictionless Signorini conditions. The behaviour of the deformable body is governed by a strain energy j and the variational formulation of the equilibrium problem concerning this deformable body under body forces f reads

$$(\mathcal{P}^\epsilon) \quad Inf \left\{ \int_{\Omega_+^\epsilon} j(e(u))dx - \int_{\Omega_+^\epsilon} f.udx, \; u_n \le 0 \text{ on } \gamma^\epsilon, \; u = 0 \text{ on } \partial_e\Omega \right\}.$$

The limit problem, when ϵ tends to 0, is investigated. To reach a result which is as explicit as possible, the asperities are assumed to be periodically

distributed. It can be proved ([6]) that $(\mathcal{P}^\epsilon)$ converges in the sense of Γ-convergence (Attouch [7]) to:

$$(\mathcal{P}^{hom}) \quad Inf \left\{ \int_{\Omega_+} j(e(u))dx - \int_{\Omega_+} f.udx + \int_{\Sigma^0} \varphi(u)ds, u = 0 \text{ on } \partial_e\Omega \right\},$$

where the new term φ on Σ^0 expresses the homogenized contact law on the "flattened" or homogenized boundary Σ^0:

$$\sigma.n^0 \in -\partial\varphi(u) \text{ on } \Sigma^0, \text{ or equivalently } \; u \in \partial\varphi^*(-\sigma.n^0).$$

The boundary energy φ will be specified in section 2.

2. CONVERGENCE RESULT

2.1 Notations and assumptions:

The following notations will be used (Figure 1) :

- Ω is a domain in $\mathbb{R}^N$ (N = 2 or 3) with a C^1 boundary. It is composed of the deformable body Ω_+^ϵ and of the rigid substrate sharing a common undulating boundary γ^ϵ with equation

$$x_N = -\epsilon h\left(\frac{x'}{\epsilon}\right), x' = (x_1, ..., x_{N-1}).$$

h is a C^1 function.

- Ω_+^ϵ is the subdomain of Ω occupied by the deformable body and Ω_-^ϵ is the subdomain occupied by the rigid substrate

$$\Omega_+^\epsilon = \left\{x \in \Omega, x_N > -\epsilon h\left(\frac{x'}{\epsilon}\right)\right\},$$

- $\Omega_+ = \{x \in \Omega, x_N > 0\}, \quad \Omega_- = \Omega - \bar{\Omega}_+, \quad \Sigma^0 = \{x \in \Omega, x_N = 0\}.$
- Q is the unit cell obtained from the real micro-unit cell by a suitable re-scaling with amplitude $\frac{1}{\epsilon}$

$$Q = \{y \in \mathbb{R}^N, y_N > -h(y'), y' \in]0,1[^{N-1}\},$$

$$\gamma^1 = \{y \in \partial Q, y_N = -h(y')\}, \gamma^0 = \{y \in Q, y_N = 0\}.$$

- $\gamma^{\#}$ denotes the vertical boundaries of Q on which the periodicity conditions will hold. A periodic (or anti-periodic) function is denoted f # (or f -#) when it has equal (or opposite) values on the opposite sides of $\gamma^{\#}$.

The constitutive law of the body occupying Ω_+^ϵ is given by a strain energy j. j can also be interpreted as a dissipation potential when u is interpreted as a velocity field instead of a displacement field. Throughout the study j is a convex and continuous function on $\mathbb{R}_s^{N\times N}$. Several cases of interest, ranging from linear elasticity to Hencky Plasticity and including Norton-Hoff materials, can be covered by assuming that j takes the form

$$j(e) = k(Tr(e))^2 + j^D(e^D), \quad (1)$$

$$\text{with } j^D(e^D) = \frac{\Lambda}{p+1}\left(\left|e^D\right|^{p+1}\right), \quad (2)$$

$$\text{and } 0 < k < +\infty, \quad 0 < \Lambda < +\infty \quad (3)$$

where e^D is the deviatoric part of e. The case $p = 1$ corresponds to linear elasticity, while the case $p = 0$ corresponds to Hencky Plasticity with Von Mises criterion.

The body is subjected to body forces $f \in L^N(\Omega)^N$ and is clamped on its outer boundary $\partial_e\Omega$ (cf Figure 1):

$$\sigma^\epsilon \in \partial j\left(e\left(u^\epsilon\right)\right), \quad \text{div}\sigma^\epsilon + f = 0 \text{ in } \Omega_+^\epsilon, \quad u^\epsilon = 0 \text{ on } \partial_e\Omega.$$

$e(u)$ is the linearized strain tensor associated with the displacement field u. The body is assumed to undergo only infinitesimal transformations. The contact between Ω_+^ϵ and the rigid substrate is governed by the frictionless Signorini conditions on γ^ϵ:

$$u_n^\epsilon \le 0, \quad \sigma_n^\epsilon \le 0, \quad u_n^\epsilon \sigma_n^\epsilon = 0, \quad \sigma_t^\epsilon = 0 \text{ on } \gamma^\epsilon,$$

$$\text{where } u_n^\epsilon = u^\epsilon . n^\epsilon, \; \sigma_n^\epsilon = \sigma_{ij}^\epsilon n_j^\epsilon n_i^\epsilon, \; (\sigma_t^\epsilon)_i = \sigma_{ij}^\epsilon n_j^\epsilon - \sigma_n^\epsilon n_i^\epsilon,$$

$$n^\epsilon \text{ outer normal vector to } \gamma^\epsilon.$$

The variational formulation of the equilibrium problem is $(\mathcal{P}^\epsilon)$.

2.2 Γ-Convergence

Define on $L^1(\Omega)^N$ the functional:

$$F^\epsilon(u) = \begin{cases} \displaystyle\int_{\Omega_+^\epsilon} j(e(u))dx, \text{when } u \in H^1(\Omega_+^\epsilon)^N, \quad u = 0 \text{ in } \Omega_-^\epsilon, \\ \qquad u_n \le 0 \text{ on } \gamma^\epsilon, \quad u = 0 \text{ on } \partial_e\Omega, \\ \qquad = +\infty, \text{ otherwise.} \end{cases}$$

Due to the possible linear growth of j, the appropriate functional space for studying this functional is BD(Ω) ([8], [9]) and the strain fields to be taken into consideration can possibly be measures. Define

$$F(u) = \int_{\Omega_+} j(e^a(u))dx + \int_{\Omega_+} j^\infty(e^s(u)) + \int_{\partial_e\Omega} j^\infty\left(-u \otimes_s n\right) ds$$

$$\text{where } j^\infty(z) = \lim_{t \to +\infty} \frac{j(tz)}{t}.$$

$e^a(u)dx$ and $e^s(u)$ denote the absolutely continuous part and the singular part of the measure $e(u)$ with respect to the Lebesgue measure ([9], [10]). j^∞ is the recession function associated with j. The functional F involves a relaxation of the boundary condition on $\partial_e\Omega$. In addition, define, when z is in $\mathbb{R}^N$

$$\varphi(z) = Inf\left\{\int_Q j^\infty(e(v))dy, \quad v \in \left(\mathcal{D}\left(\bar{Q}\right)\right)^N, \right.$$

$$\left. v\# \text{ on } \gamma^\#, \quad v_n(y) + z_n \le 0 \text{ on } \gamma^1 \right\}. \quad (4)$$

Theorem: *Under the above assumptions, F^ϵ Γ-converges into $L^1(\Omega)^N$ strong to F^{hom} :*

$$F^{hom}(u) = \begin{cases} F(u) + \displaystyle\int_{\Sigma^0} \varphi(u^+)ds \text{ when } u \in BD(\Omega), \quad u = 0 \text{ on } \Omega_-, \\ \qquad = +\infty \text{ otherwise.} \end{cases}$$

u^+ is the trace of u on Σ^0 into Ω_+ .

Corollary: *Assume that $f \in L^N(\Omega)^N$. Then $(\mathcal{P}^\epsilon)$ converges to $\left(\mathcal{P}^{\text{hom}}\right)$.*

The properties of φ are detailed in the following proposition.

Proposition: *φ is a convex function which is lower-semi-continuous and positively homogeneous to the degree one on $\mathbb{R}^N$. It is the support function of the convex set K:*

$$K = \left\{ T \in \mathbb{R}^N, \exists \ \sigma \in \mathcal{K}, \ T = -\int_{\gamma^0} \sigma.n^0 ds \right\}$$

$$\textit{where} \ \ \mathcal{K} = \Big\{ \sigma \in L^\infty(Q)_s^{N \times N}, \ \ \sigma \in \mathrm{dom} j^*, \ \ \mathrm{div}(\sigma) = 0,$$

$$\sigma.n \quad -\# \ \text{ on } \ \gamma^\#, \ \ \sigma_n \leq 0 \ \text{ and } \ \sigma_t = 0 \ \text{ on } \ \gamma^1 \Big\}$$

3. RESULTS AND DISCUSSIONS

3.1 Preliminary comments

1.The contact law between the deformable body and the rigid substrate is governed by the relations:

$$\sigma.n^0 \in -\partial\varphi(u) \ \text{ on } \ \Sigma^0, \ \text{ or equivalently } \ u \in \partial I_K(-\sigma.n^0),$$

where n^0 is the outer normal vector to Σ^0. The traction $\sigma.n^0$ on Σ^0 is constrained to stay in a convex set $-K$, in agreement with classical friction laws, e.g. Tresca's or Coulomb's laws. However the displacement is an outer normal vector to the set of admissible tractions. This associated "flow rule" is in agreement with Tresca's law but not with Coulomb's law, which is nonassociated.

2. When $p > 0$, it can readily be checked that $j_\infty = I_{\{0\}}$. Therefore

$$\varphi(z) = I_\Gamma, \ \text{ where } \ \Gamma = \left\{ z \in \mathbb{R}^N, \ \ z.n(y) \leq 0 \ \ \forall y \in \gamma^1 \right\}.$$

K is Γ's polar cone:

$$K = \Gamma^* = \left\{ T \in \mathbb{R}^N, \ \ T.z \leq 0 \ \ \forall z \in \Gamma \right\}.$$

Γ and Γ^* are directly correlated with the shape of the asperities and the determination of the contact law is reduced to a purely geometrical problem.

In the simple case considered in Figure 2, the cones Γ and Γ^* can be given in an explicit form:

$$\Gamma = \left\{ z \in \mathbb{R}^N, \ \ |z_t| \leq -\mathrm{cotg}(\theta) z_n \right\}, \ \ \Gamma^* = \left\{ T \in \mathbb{R}^N, \ \ |T_t| \leq \mathrm{tg}(\theta) T_n \right\}$$

and the contact law reads

$$|\sigma_t| \leq -\mathrm{tg}(\theta)\sigma_n, \ \ u_t = -\lambda \frac{\sigma_t}{|\sigma_t|}, \ \ u_n = -\lambda \mathrm{tg}(\theta),$$

$$\lambda \geq 0, \ \lambda = 0 \ \text{ when } \ |\sigma_t| < -\mathrm{tg}(\theta)\sigma_n.$$

(u_t, u_n) and (σ_t, σ_n) denote the tangential and normal components of u and $\sigma.n$ on Σ^0 ($u_n = u.n^0 = -u_N$, $\sigma_n = \sigma.n^0.n^0$). The homogenized contact law specifies that except when $p = 0$, a ***dilatancy*** effect is to be observed: a non vanishing tangential displacement is accompanied by a nonvanishing negative *normal displacement* of the contacting surface. This normal displacement should not be interpreted as a loss of contact but as the motion of asperities in the deformable body moving over asperities in the rigid substrate.

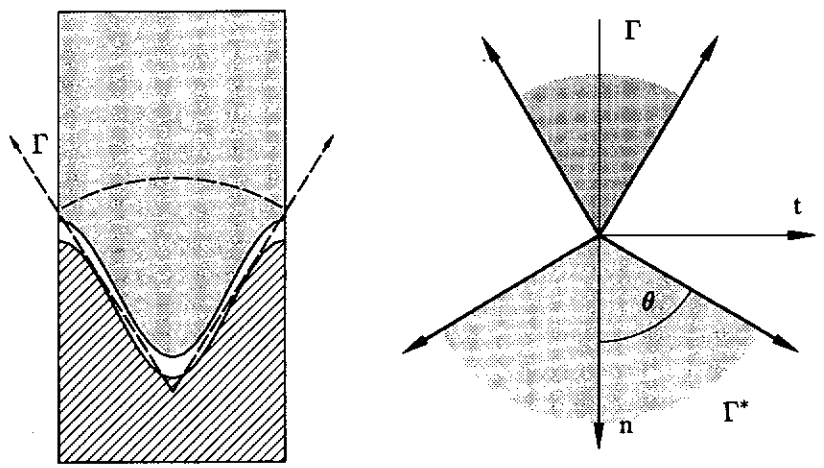


Fig. 2: Γ and Γ^*, cones of admissible displacements and tractions

3. When $p = 0$, the set K ceases to be a cone but the contact law remains an associated law. A few simple estimates of K can be derived from the definition of φ. When j takes the form (1):

$$\varphi(z) = +\infty \quad \text{when} \quad z.n^0 \geq 0$$

In other terms, the possibility of interpenetration occuring between the deformable body and the rigid substrate is excluded. This physically sound result follows from the incompressibility constraint, which implies that

$$j^\infty = +\infty \quad \text{when} \quad \text{div}(u) \neq 0.$$

Further upper bounds on φ can be obtained with specific choices of v. When $z \in \Gamma$ the following discontinuous displacement fields can be considered

$$v(y) = -z \quad \text{when} \quad y_N \leq h(y'), \quad v(y) = -z' \quad \text{when} \quad y_N > h(y').$$

The corresponding estimate on φ reads:

$$\varphi(z) \leq Inf\left\{ j^\infty\left(n^0 \otimes_s (z' - z)\right), \quad z' \in \Gamma \right\}.$$

3.2 Numerical determination of K

When $p \neq 0$ the determination of the convex set Γ amounts to a purely geometrical problem; whereas when $p = 0$, K has to be determined numerically. The variational problem (4) is a limit analysis problem posed on the unit cell Q with nonclassical boundary conditions (periodicity conditions) and a nonclassical loading (z is specified). The situation is similar to that encountered in the homogenization of ideally plastic materials. Modifications which have to be applied to classical limit analysis algorithms in the latter context have been discussed in Michel and Suquet [12]. Similar modifications are required when dealing with boundary homogenization. In the results discussed below, the limit states were obtained by solving an evolution problem for an elastic ideally plastic material occupying Q. An additional difficulty arises from the fact that Q is infinite is the direction of y_N. For numerical purposes, Q was truncated at $y_N = R$ and the boundary condition at infinity was replaced by $v = 0$ when $y_N = R$. When R was large enough, it was checked to ensure that the results were insensitive to the choice of R. Calculations were performed under plane strain conditions. The unilateral condition on γ^1 was dealt with by means of an interface finite element and the non interpenetration constraint was approximately

satisfied by adopting a penalty method. The Von Mises criterion was adopted in order to model the plastic behaviour of the deformable body and the corresponding recession function j^∞ reads

$$j^\infty(\epsilon) = \sigma_0 \epsilon_{eq}, \quad \text{with} \quad \epsilon_{eq} = \left(\frac{2}{3}\epsilon_{ij}\epsilon_{ij}\right)^{1/2}.$$

σ_0 is the yield stress of the material constituting the asperities.

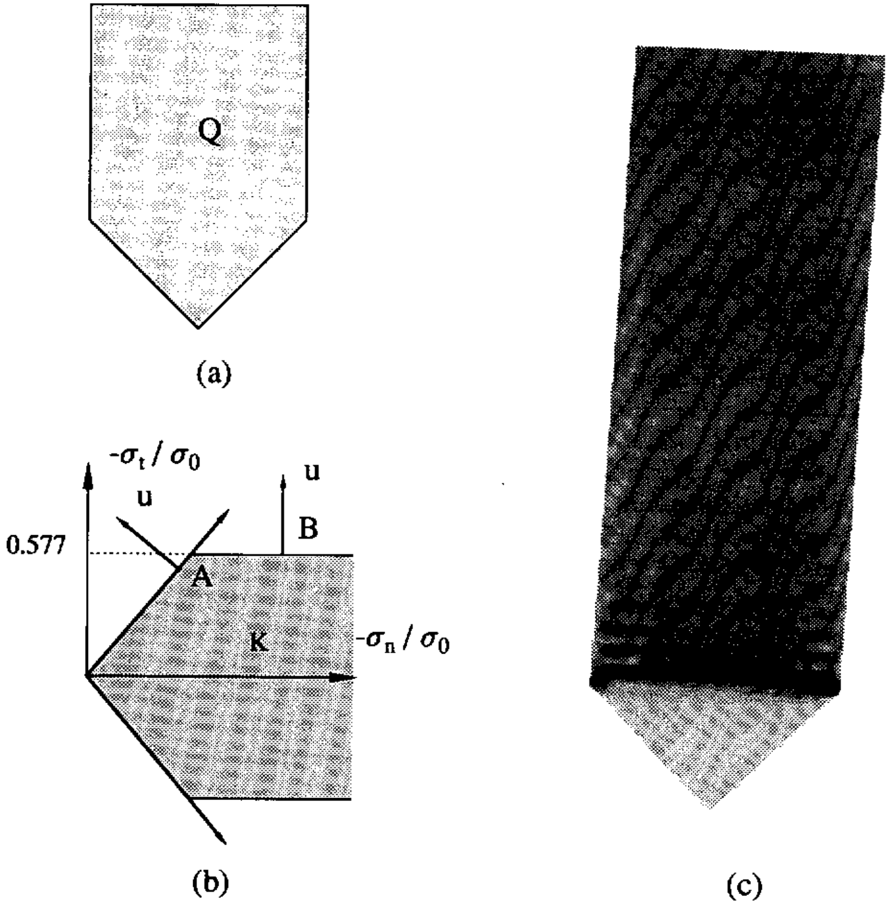


Fig. 3: (a) Geometry of the asperity. (b) Set K and associated rule. (c) Failure mode with no dilatancy (point B). Deformed configuration

It should be noted that plasticity entails a limitation of the shear component of the stress vector on Σ^0, and this remark is in agreement with Orowan's modification of the classical Coulomb-Amonton law (see *e.g.* Wanheim *et al* [3]. For the geometry of the asperity under consideration, the limit under shear conditions is $\sigma_0/\sqrt{3}$, i.e. exactly the limit under shear of the material of which the asperities are composed. Orowan has suggested a friction model in which the friction shear is proportional at low pressures to the normal pressure and is equal at high pressures to the yield stress of the material composing the asperities under pure shear ([3]). Two mechanisms are involved in these two regimes, as shown in Figure 3. This result can be obtained directly with a simple plastic mechanism using a slip line theory approach, with a rigid motion of the upper part of the unit cell and no motion of the asperity itself. This simple mechanism is in agreement with the mechanism observed numerically.

3.3 Limitations of the model

The above model gives good sthenic results (i.e. it predicts the tractions satisfactorily), but the kinematic predictions as to the dilatancy, seem to be less physically realistic, at least at first sight, and require re-examining the hypotheses underlying the model. The model has several variably severe limitations.

i) The first limitation of the model is the periodicity imposed on the distribution of the asperities, so that in the case of the local problem, it was possible to focus on a single deterministic unit cell. This assumption is less restrictive than it may appear. From the theoretical standpoint, it has been used to obtain a problem which is clearly stated and amenable to a mathematical study, providing clear evidence that the parameter involved is small. The structure of the result (i.e. the convexity of φ which indicates that the contact law is associated) would be similar given a more general description of the contact surface. From the numerical standpoint, the assumption of periodicity leads to unambiguous boundary conditions on the cell. It would not be difficult to replace these boundary conditions by others, provided that the new boundary conditions could be properly correlated with a statistical description of the contact surface. As regards how to describe the surface, what would be of real interest would be to develop a *stochastic* boundary homogenization approach. This is a considerable task for nonlinear problems. Returning to geometric considerations, it has been assumed in this model that the asperities of the deformable body and those of the substrate match exactly. Although we did not investigate this point, it should not be difficult to investigate two undulating periodic boundaries with the same wavelength but with different profiles. *Contractancy* (i.e. close intrication of the asperities of both surfaces) as well as dilatancy are to be expected in this case. The stochastic case is (again) a difficult open problem.

ii) The second limitation with this approach is the quasi-static approximation. The energy of elastic waves emanating from the contact surface and propagating into the solid is "lost" by the small layer and should contribute in the limit $\epsilon \to 0$ to the dissipation of the contact zone. It has been implicitly assumed here that this dissipation is small in comparison with the plastic dissipation of the asperities. To the best of our knowledge, this seems to be the rule with metallic materials.

iii) The next, and probably most serious, limitation of the present approach is that it is based on an equilibrium problem and not on an evolution problem.

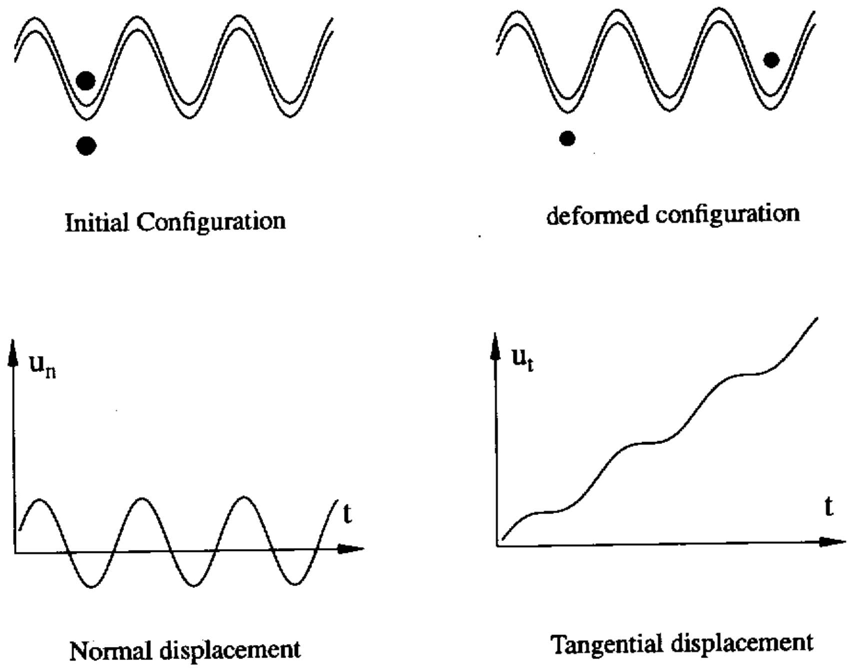


Fig. 4: Large displacements of asperities.

In other words, the problem has to be set in terms of velocities, rather than in terms of displacements, and solved within a finite time interval with due attention to the displacements, which may be of the order of magnitude of ϵ at the local scale. Hence a formulation involving the velocity as the principal unknown and allowing for large displacements, at least of the order of ϵ (which are large displacements at the level of the asperities), should be used. It is expected that the normal and tangential displacements under a constant stress will involve oscillations corresponding to the relative motions of the one surface over the other. Both the amplitude and the period of the oscillations involved in the normal displacement are of the order of ϵ, and the normal displacement vanishes on average within a finite (non vanishing) time interval; whereas the tangential displacement oscillates around a linearly increasing average value (the tangential velocity therefore oscillates around an average constant value). The mechanical formulation and the mathematical resolution of an evolution problem of this kind are beyond the present possibilities of the authors.

4. Mechanistic models

Two simple "mechanistic" models were considered to account for small scale configuration changes. The first is based on the dilatancy effect established with the above mathematical model. The second model which was inspired by the "Critical state theory" of the Cambridge school, is not directly connected to boundary homogenization but again involves taking dilatancy to be a central variable in the understanding of friction. Both models focus on incipient motions rather than on large slips and are based on associated laws. The dilatancy of the layer between the bodies in contact plays the role of an internal variable describing the mechanical state of the contact surfaces.

4.1 A model with a two-fold mechanism

In the first model, successive configurations are considered and the unknown u, which was previously interpreted as being either a displacement or a velocity field, is now definitely a *velocity field*. The evolution is viewed as a sequence of equilibriums to which the above reasoning is applied. To simplify, the model is presented here only for piecewise straight asperities,

as depicted in Figure 5.

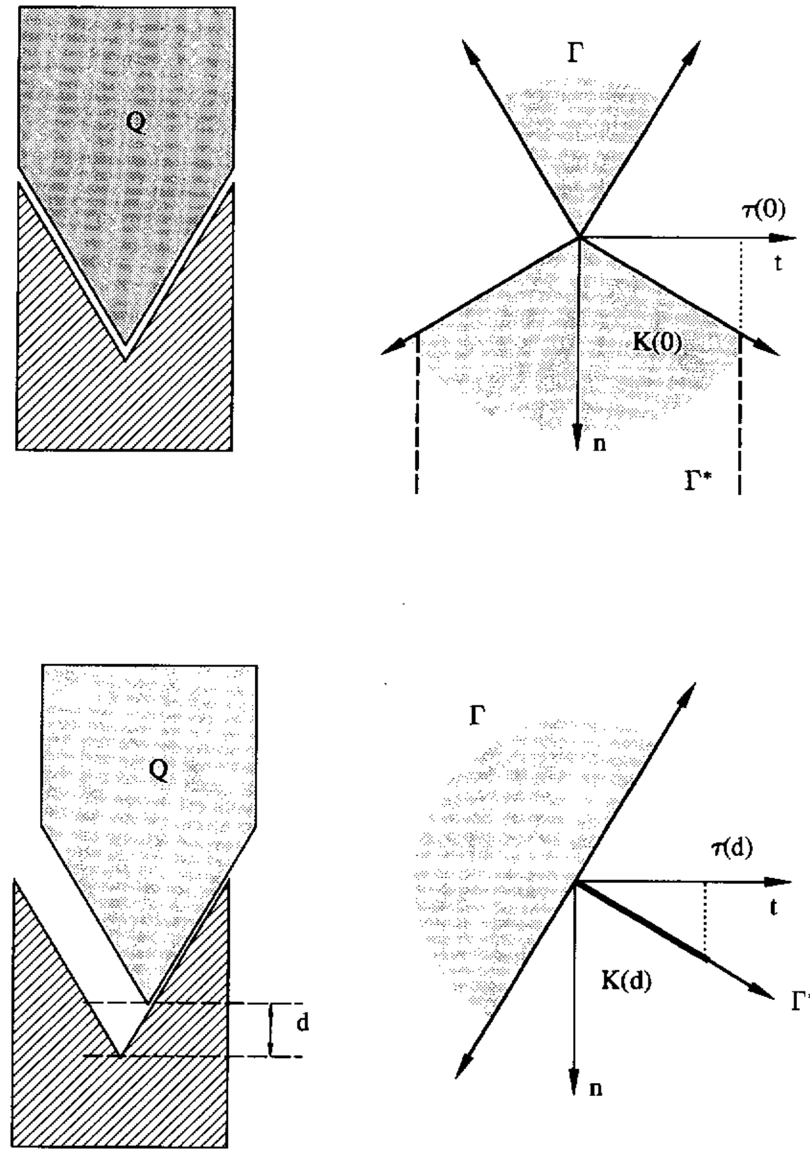


Fig. 5: Motion of asperities. A simplified model

Changes in configuration are expressed in terms of a scalar parameter d, namely the distance between a plane materially attached to the deformable body and a plane materially attached to the rigid substrate. Both planes are assumed to coincide when $d=0$. When $d \neq 0$, the above reasoning can be reproduced and a convex set $K(d)$ of admissible forces can be defined. For the unit asperity considered in Figure 5, $\Gamma^*(d)$ is a half-line and $K(d)$ a segment. d acts as a softening variable: the limit under shear conditions of the homogenized material, which is implicitly contained in $K(d)$, is a decreasing function of d. Assume now that the deformable body is in the initial configuration ($d=0$ and subjected to a force such that the shear stress $|\sigma_t|$ is proportional to the normal pressure $-\sigma_n$, but below the shear strength predicted by $K(0)$. The kinematic mechanism activated first is the dilatancy and the two surfaces begin to separate. d increases while $K(d)$ decreases. Since the overall force remains constant, the shear strength and σ_t will coincide at some value of d and the plastic flow mechanism will consequently be activated. The resulting motion is possibly purely tangential. In conclusion, there exists a time interval during which the dilatancy prevails. Afterwards, the prevailing mechanism will be the plastic flow of asperities.

The evolution equation for the pair of unknowns (u, d) reads:

$$-u \in \partial I_{K(d)}(\sigma.n), \quad \dot{d} = -u_n.$$

An even simpler and more explicit model can be proposed:

$$|\sigma_t| \le -\mu\sigma_n, \quad |\sigma_t| \le \tau(d), \quad \tau(d) = \frac{\sigma_0}{\sqrt{3}}\left(1 - \frac{d}{h}\right) \tag{5}$$

$$-u_t = (\lambda_1 + \lambda_2)\mathrm{sign}(\sigma_t), \quad -u_n = \lambda_1\mu, \quad \dot{d} = -u_n.$$

λ_1 and λ_2 are the two positive multipliers corresponding to the dilatant mechanism and the shear mechanism associated with the constraints (5). μ and h are the two material parameters of the model correlated with the shape of the asperities and with their height.

4.2 Elliptic model

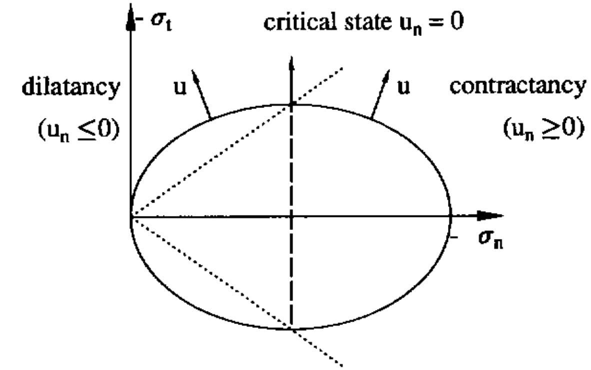
The second model was inspired by the "Critical state theory" developed for frictional materials (clay, sand, etc) by the Cambridge school [13]. An elliptic surface is considered

$$\left(\frac{\sigma_t}{\mu}\right)^2 + \left(\sigma_n + \frac{p}{2}\right)^2 \le \left(\frac{p}{2}\right)^2, \tag{6}$$

and the normality rule is adopted

$$u_t = -\frac{2\lambda}{\mu^2}\sigma_t, \quad u_n = -2\lambda\left(\sigma_n + \frac{p_c}{2}\right), \quad \dot{d} = -u_n, \quad \lambda \ge 0. \tag{7}$$

The sign of $\left(\sigma_n + \frac{p_c}{2}\right)$ delimits the region of dilatancy $u_n \le 0$ from the region of contractancy $u_n \ge 0$, while the Coulomb's cone is the locus of "critical states" ($u_n = 0$).



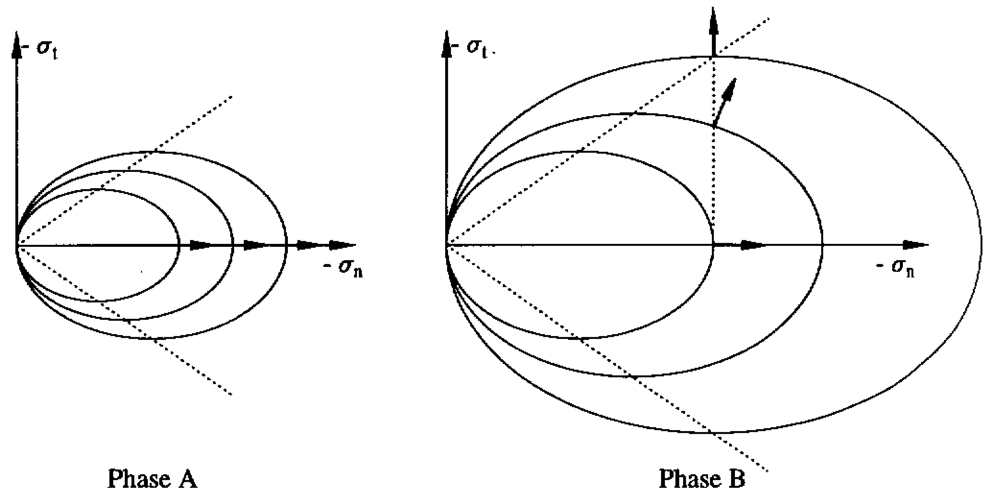


Fig. 6: Elliptic model. Response along a path of applied normal pressure followed by a path of applied shear.

The pressure p_c is related to the distance d between the two contacting bodies by a consolidation relation. A simple example of this relation is:

$$p_c = p_0 \exp(-K(d-h)). \tag{8}$$

It is instructive to examine the response of the model to a path of imposed normal pressure followed by a shear (Figure 6). During phase A, where the normal pressure is progressively increased, u_n is positive and d decreases, i.e. the contact between the two bodies becomes more intricate. The consolidation pressure increases and the response of the model is stable and hardening. In phase B, where the shear force is applied, the contractancy of the layer decreases up to a critical state where it vanishes. At this point the shear cannot be increased while the normal pressure is maintained, since (σ_t, σ_n) would be in the region of dilatancy which from (6)(7)(8) would yield an increase in d, i.e. a decrease in the ellipse size, in contradiction with the increase in the applied shear. The Coulomb's cone is therefore interpreted as being the locus of critical or unstable states. The present model involves four material parameters. μ and h are correlated with the shape of the asperities, while p_0 and k, which are not justified here by micromechanical arguments, have to be measured directly. p_0 is the normal pressure under which the asperities yield in the initial configuration.

Conclusions

A mathematical model based on the homogenization of boundaries with small asperities is presented. The model predicts a possible dilatancy of the layer surrounding the contact surface. It is not only due to shortcomings but is also physically relevant at least in the earliest stages of the relative motions of the contacting bodies. Two simplified models are proposed to incorporate the dilatancy as a softening variable. More attention should be paid in future studies to the effects of contractancy or dilatancy in the thin layer involved in the contact.

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