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Identification of Shear Wave Parameters of Viscoelastic Solids by Laboratory Measurements of Stoneley-Scholte Waves

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Summary
This paper deals with the problem of viscoelastic solid characterization by acoustical means and in particular with the recovery of the shear wave parameters. It has been previously shown, in the Underwater Acoustics field, that the shear wave parameters of the sea floor could be recovered by using inverse techniques applied to propagation characteristics of interface waves such as Stoneley-Scholte waves, which can propagate in water/sediment configurations. The goal of the study presented in the paper is then to test models, commonly used for seabed identification, on media whose properties are well-controlled by laboratory tank experiments, contrary to in situ bottoms. It is shown that the viscoelastic medium parameters can also be identified from the characteristics of interface waves, generated experimentally in laboratory on very attenuating synthetic materials. The paper presents results about the estimated shear wave parameters obtained from both numerical and experimental data by applying Brent's method on the characteristics of the interface waves. The observation and the discussion of differences between theoretical and experimental results are the goal of the paper. The study presented here validates the forward model previously developed and it can be considered as a first step towards the direction of acoustic classification of sea bottoms.
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1. Introduction

The physical properties of sea floor sediments is of great interest for marine geologists, people working in the underwater acoustics domain as well as people involved in oil exploitation or in industrial fisheries. The applications cover the field of marine environment, geotechnical research, civil engineering and underwater acoustics. Usual techniques (direct in situ techniques, acoustical probing...) lead to the determination of the compressional wave parameters and the densities of the layers [1, 2], and sometimes to the determination of the shear wave velocity of the sediments [3]. Over the past two decades an increasing effort has been put into this field. Recent developments in forward and inverse modeling indicate that the acoustic and seismic waves in sea floor can be used for remote sensing of the geotechnical properties of sediments. In particular the propagation of interface waves observed in such an environment is strongly dependent on the physical characteristics of sea bottom. Since the velocity of both Love waves and Stoneley-Scholte waves, called also Scholte waves, is strongly linked to the shear wave velocity of the sedimentary bottom, one can easily obtain information about the shear wave velocity of sediments by applying inverse techniques to the dispersion curves of interface waves [4, 5, 6, 7]. Few works are concerned with the determination of the shear wave attenuation in the sedimentary layer [8].

Even if the different models commonly used for identification of sediment properties seem to be reliable, a question may be however raised: how to be sure of the accuracy of the estimated parameters of non-controlled media? The aim of the paper is then to give an answer to the question by considering the possibility of recovering the well-controlled solid parameters with the propagation characteristics of Stoneley-Scholte waves in water/absorbing solid configurations. The solid medium was represented by attenuating synthetic resins and the interface wave properties were obtained from experiments performed in laboratory.

The first author has previously conducted theoretical and experimental studies of the Stoneley- Scholte wave propagation in water/viscoelastic solid configurations [9, 10]. The originality of the studies was to release the assumption of non absorbing media and to suppose the solid bottom to be highly viscoelastic. The medium was represented by a Kelvin-Voigt solid, which is generally described by a parallel connection of stiffness and viscosity elements. The viscosity coefficients introduced in the constitutive law were then connected to the plane wave attenuations in the solid which can easily be determined by acoustical measurements. Analysis of the interface wave propagation was developed by using the approach of inhomogeneous plane waves which is really suitable for such a study. It enables one to locally describe the interface wave as a linear combination of inhomogeneous plane waves which satisfy boundary conditions at the interface in each medium. It was shown that the wave is dispersive, that the energy is generally concentrated in the fluid medium and that it decreases with increasing distance from the interface [9]. The originality of the study was to compute the velocity of the interface wave and the damping of its energy along its propagation path at the water/viscoelastic solid interface. These properties were also verified experimentally in laboratory with absorbing synthetic resins [10], which demonstrates the validity of the theoretical model previously developed. Some of the propagation characteristics of Stoneley-Scholte wave are reviewed in Appendix at the end of the paper.

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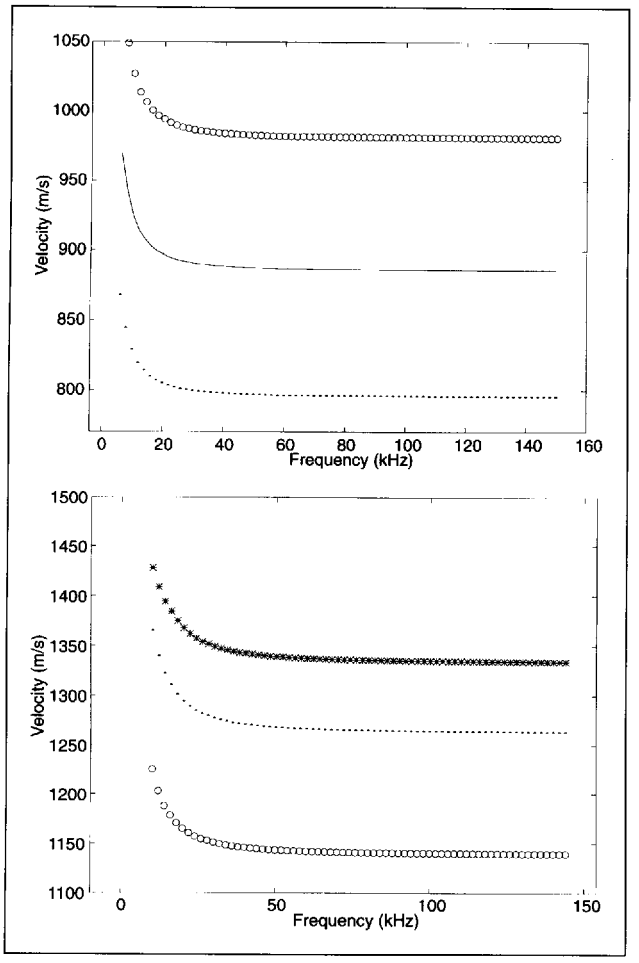

Figure 1. Effect of the changes in the shear wave velocity of the viscoelastic medium on the dispersion curves of the phase velocity of the Stoneley-Scholte wave. (a) At the water/PVC interface; ——, dispersion curves corresponding to the exact values of the shear wave velocity of PVC material, · · ·, dispersion curves corresponding to a decrease of the shear wave velocity by 15%, oooo, dispersion curves corresponding to an increase of the shear wave velocity by 15%. (b) At the water/B2900 interface; · · ·, dispersion curves corresponding to the exact values of the shear wave velocity of B2900 material, oooo, dispersion curves corresponding to a decrease of the shear wave velocity by 15%, * * *, dispersion curves corresponding to an increase of the shear wave velocity by 15%.

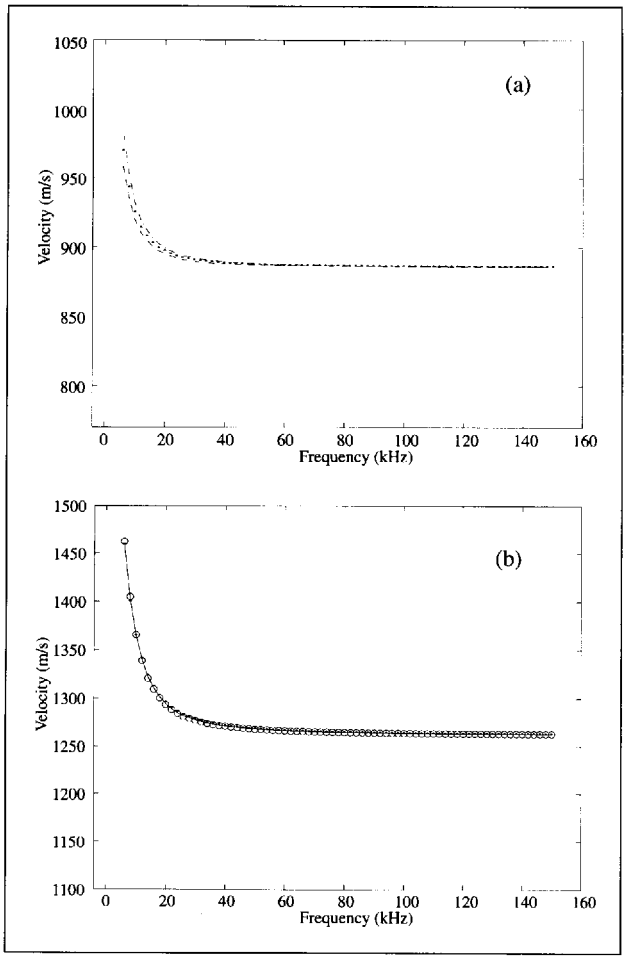

Figure 2. Effect of the changes in the shear wave attenuation of the viscoelastic medium on the dispersion curves of the phase velocity of the Stoneley-Scholte wave. (a) At the water/PVC interface; · · ·, dispersion curves corresponding to the exact values of the shear wave attenuation of PVC material, – · – ·, dispersion curves corresponding to a decrease of the shear wave attenuation by 15%, – – –, dispersion curves corresponding to an increase of the shear wave attenuation by 15%. (b) At the water/B2900 interface; ——, dispersion curves corresponding to the exact values of the shear wave attenuation of B2900 material, · · ·, dispersion curves corresponding to a decrease of the shear wave attenuation by 15%, oooo, dispersion curves corresponding to an increase of the shear wave attenuation by 15%.

Before solving the inverse problem, that is the determination of the shear wave parameters of the viscoelastic solids from the propagation characteristics of the interface wave, a parametric study of the influence of the physical properties of solids on these characteristics was necessary. Section 2 presents the results of this parametric study. These findings were then used to develop an inversion strategy.

Finally, Section 3 is devoted to the solution of the inverse problem. Inversion of interface wave data was carried out using Brent's algorithm. The inverse scheme was able to recover the shear wave velocity and attenuation of the bottom assuming an a priori knowledge of the environment. It also assumed horizontally layered media (plane and parallel interfaces) and homogeneous layers. Several test results are presented in this section. The originality of the study is that the inverse algorithm, applied to either dispersion or energy curves, was tested against both noisy synthetic data and *data obtained from experiments* performed in laboratory. The observation and the discussion of differences between theoretical and experimental results in the latter case may thus be a good criterion for the validity and applicability of the scheme performed under in situ conditions.

2. Introduction to inverse problem

It is evident from previous works on forward modeling [9, 10, 11] that the parameters of the shear waves in viscoelastic medium conditione the propagation characteristics of the Stoneley- Scholte wave at the ideal fluid/viscoelastic

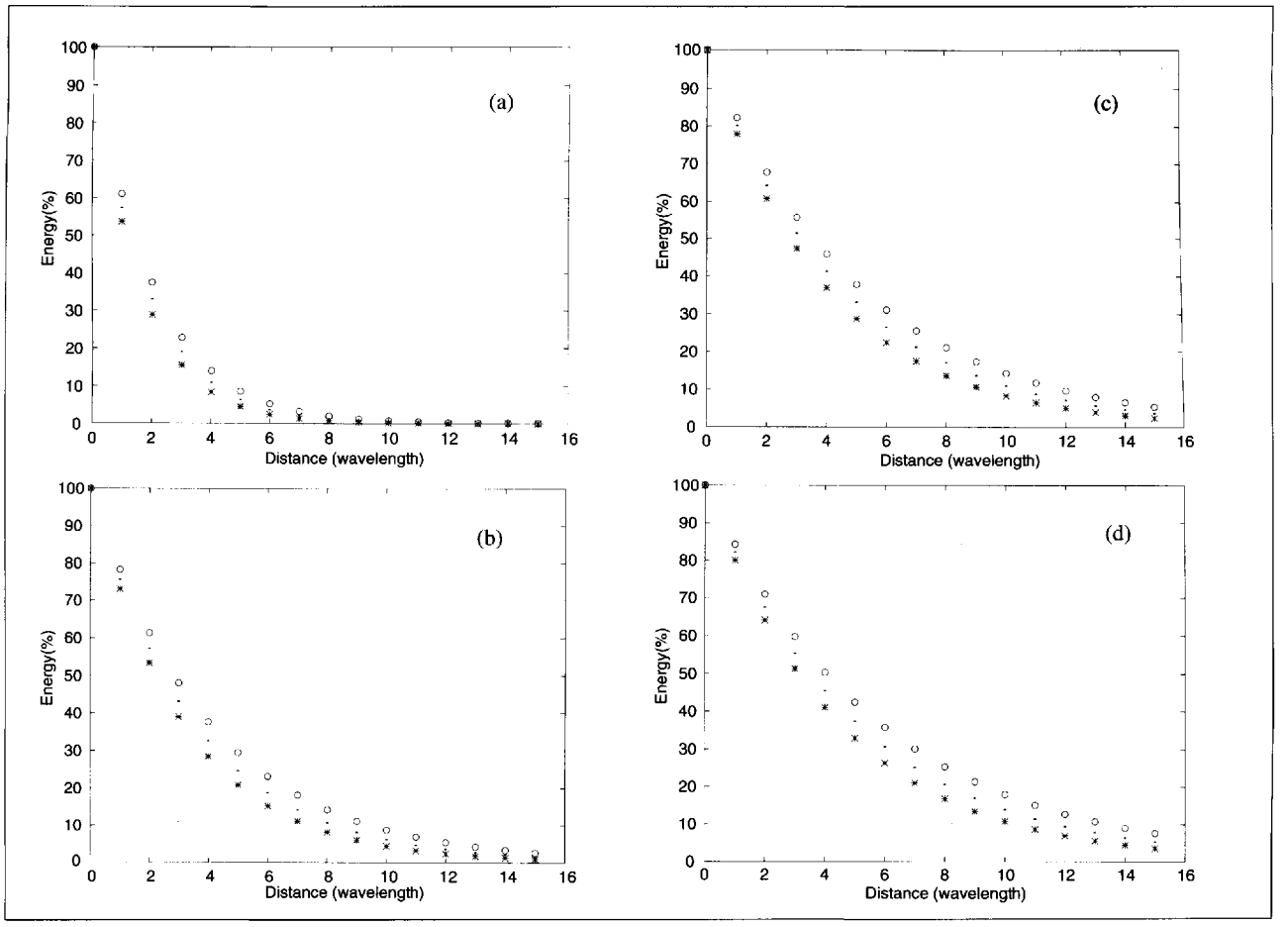

Figure 3. Effect of the changes in the shear wave attenuation of the viscoelastic medium on the energy curves of the Stoneley-Scholte wave at the water/PVC interface. (a) Frequency 30 kHz, wavelength $\lambda = 3$ cm, (b) frequency 80 kHz, wavelength $\lambda = 1.1$ cm, (c) frequency 130 kHz, wavelength $\lambda = 0.7$ cm, (d) frequency 200 kHz, wavelength $\lambda = 0.4$ cm. · · ·, energy curves corresponding to the exact values of the shear wave attenuation of PVC material, oooo, dispersion curves corresponding to a decrease of the shear wave attenuation by 15%, * * * *, dispersion curves corresponding to an increase of the shear wave attenuation by 15%.

solid interface. In particular, if the wave attenuations in the solid bottom are taken into account, the interface wave is dispersive and is attenuated during its propagation at the interface (see Appendix). The goal of this section is to report a sensitivity study, based on the forward model (see Appendix), and to determine which parameter can be recovered by inversion methods. In fact what is important to know here is the sensitivity of the propagation characteristics of the interface wave to small changes in the shear wave velocity or in the attenuation of the viscoelastic medium.

2.1. Effect of the changes in the shear wave parameters on the dispersion curves

Figures 1 and 2 illustrate the dispersion curve of the phase velocity of the Stoneley-Scholte wave in different fluid/viscoelastic solid configurations, and in particular at the water/PVC interface and at the water/B2900 interface (see Appendix).

Figure 1 corresponds to an increase and a decrease of the shear wave velocity by an amount of 15%, in comparison with the exact values. In both cases, the shear wave velocity is an important parameter for the dispersion curves: a small variation of the velocity results in a variation of about 10% of the dispersion curve of the Stoneley-Scholte wave.

Figure 2 represents variations of the dispersion curves of the Stoneley-Scholte wave when the nominal shear wave attenuation of the two solids is changed by factors of 15%. The behavior of the dispersion curves shows that the phase velocity of the interface wave is not sensitive to changes in the shear wave attenuation. Therefore shear wave attenuation can be considered as a passive parameter with respect to the dispersion of the interface wave, which has been observed in many other previous works. As the interface wave is sensitive to the shear wave velocity of the viscoelastic solid, this parameter will be estimated accurately from the dispersion curve analysis, contrary to the shear wave attenuation.

2.2. Effect of the changes in the shear wave attenuation on the energy curves

The curves illustrated by Figures 3 and 4 represent the decrease of the energy of the Stoneley- Scholte wave during its propagation in different fluid/viscoelastic solid configura-

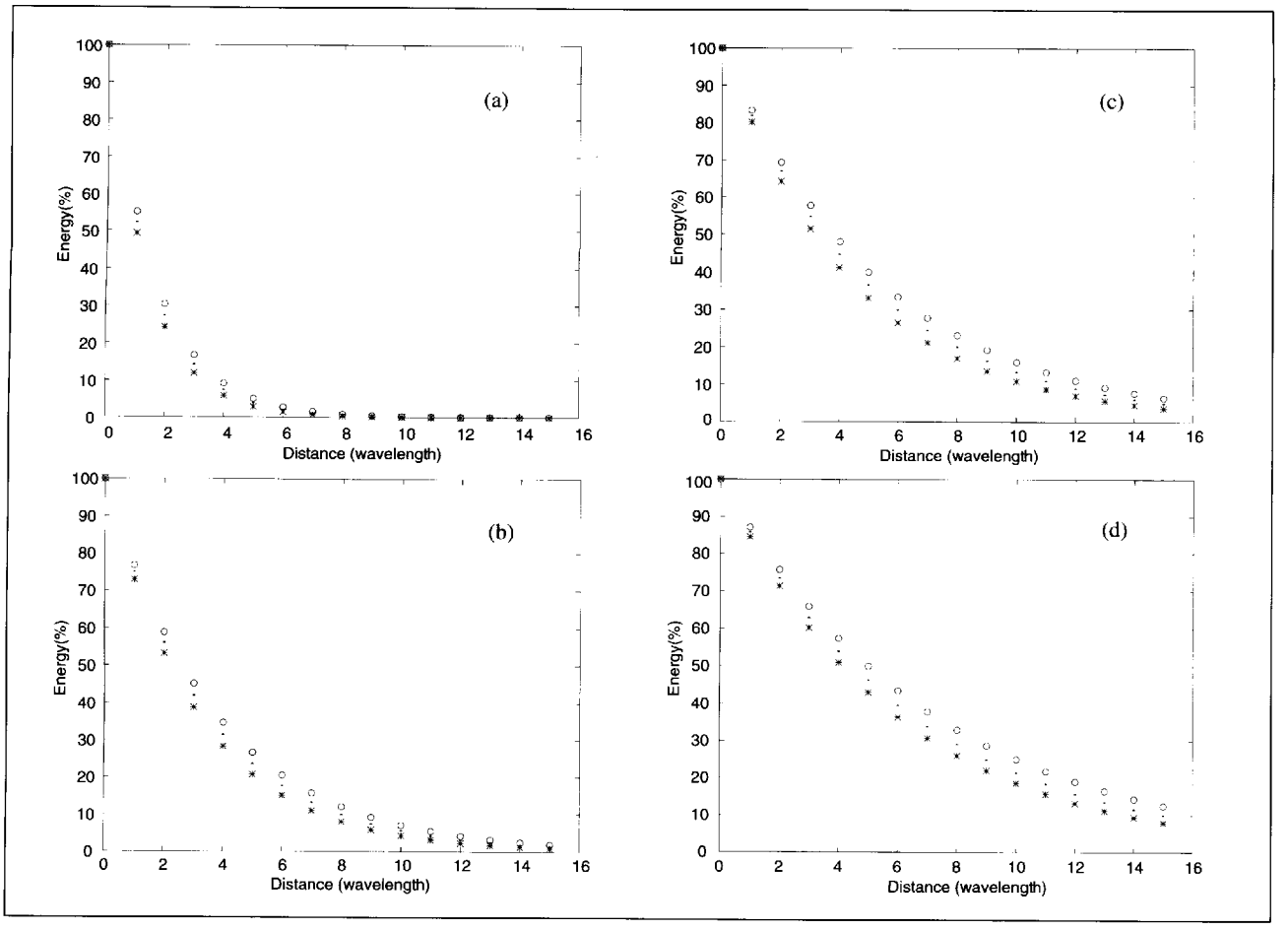

Figure 4. Effect of the changes in the shear wave attenuation of the viscoelastic medium on the energy curves of the Stoneley-Scholte wave at the water/B2900 interface. (a) Frequency 30 kHz, wavelength λ = 4.3 cm, (b) frequency 80 kHz, wavelength λ = 1.6 cm, (c) frequency 130 kHz, wavelength λ = 0.7 cm, (d) frequency 200 kHz, wavelength λ = 0.4 cm. · · ·, energy curves corresponding to the exact values of the shear wave attenuation of B2900 material, oooo, dispersion curves corresponding to a decrease of the shear wave attenuation by 15%, * * * *, dispersion curves corresponding to an increase of the shear wave attenuation by 15%.

tions, particularly at the water/PVC interface and at the water/B2900 interface. The energy curves of the interface wave, for the frequencies 30 kHz, 80 kHz, 130 kHz and 200 kHz, correspond to an increase and a decrease of the shear wave attenuation by an amount of 15%, in comparison with the exact values. The variations between the values and the exact ones are then approximately equal to 4% for each frequency and for each configuration.

The propagation characteristics of the Stoneley-Scholte wave are much less sensitive to the changes in the shear wave attenuation of the viscoelastic medium than to the ones in the shear wave velocity. Nevertheless the estimation of the attenuation will be accurate enough for a rough identification of the solid properties.

3. Inverse problem

The problem under consideration is the determination of the shear wave parameters of viscoelastic solids from both theoretical and experimental propagation characteristics of Stoneley- Scholte waves propagating at ideal fluid/viscoelastic solid interfaces. The observation and the discussion of differences between theoretical and experimental results are the goal of the section.

3.1. Some definitions

It is well-known that the propagation characteristics of interface waves are fairly insensitive to density and compressional parameters of media and that they are function of shear wave parameters of solids only. In the previous section, the effect of small changes in the shear wave parameters of homogeneous materials on the characteristics of the Stoneley-Scholte wave has been studied. From these results it can then be concluded that it is possible to determine *separately* the shear wave velocity and the shear wave attenuation of the medium from the dispersion curves of interface waves and their energy curves. Techniques for estimating the shear wave velocity of solids generally consist in measuring the dispersion of interface waves at different frequencies and using this information to recover the unknown parameter. In a similar way, the shear wave attenuation of solids can also be inferred from the estimation of the energy loss of interface waves

Table I. Estimation of the shear wave velocity c_t of PVC and B2900 materials from noisy data issued from dispersion curves of the Stoneley-Scholte wave at the water/solid (PVC or B2900) interface for different initial estimates of c_t. (In the PVC case the actual c_t is equal to 1100 m/s, and in the B2900 case the actual c_t is equal to 1620 m/s.)

	Initial value of c_t (m/s)	0.75 c_t	0.85 c_t	0.95 c_t	1.05 c_t	1.15 c_t	1.25 c_t	1.35 c_t	1.45 c_t	1.55 c_t	1.65 c_t
PVC	estimate of c_t (m/s)	1098	1104	1091	1095	1097	1098	1103	1099	1096	1103
	estimation errors					< 1%					
B2900	estimate of c_t (m/s)	1686	1621	1580	1694	1639	1551	1532	1552	1558	1532
	estimation errors	4%	<1%	2.5%	4.6%	1%	4.3%	5.4%	4.2%	3.8%	5.4%

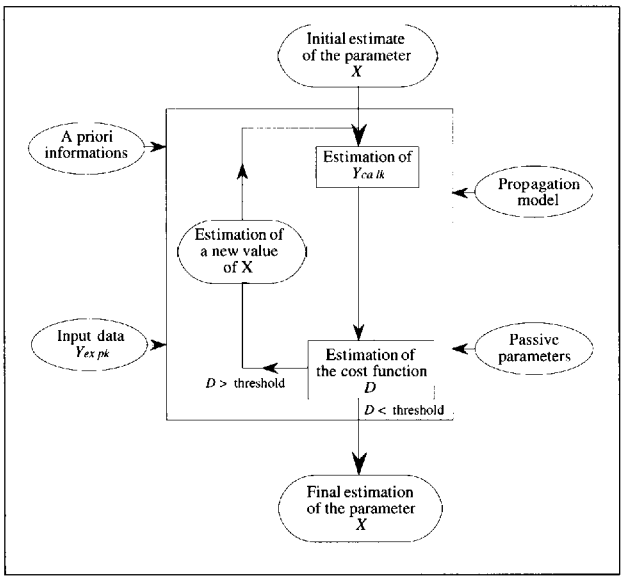

Figure 5. The schematic diagram of the inverse scheme.

during their propagation, as it will be shown in the second part of the section. The relationship between the N measured data $\boldsymbol{Y}_k$ ($0 \leq k \leq N$) (i.e. phase velocities $c_{\phi k}$ of interface wave, respectively amount of energy lost E_k), function of the quantities $\boldsymbol{W}_k$ (i.e. frequencies f_k, respectively distances of propagation d_k), and the unknown parameter $\boldsymbol{X}$ (i.e. shear wave velocity c_t, respectively shear wave attenuation α_t of solids) can be written in the form of a non linear system of N equations:

$$\boldsymbol{F}(\boldsymbol{W}_k, \boldsymbol{Y}_k, \boldsymbol{X}) = \boldsymbol{0}, \qquad 0 \leq k \leq N, \tag{1}$$

where $\boldsymbol{F}$ relates the observed data to the unknown parameters. $\boldsymbol{F}$ can be the dispersion relation or the function expressing the decrease of energy of interface waves during their propagation. The sensitivity analysis of Section 2 clearly shows that this problem is well-posed. Different homogenous materials give origin to different dispersion or energy curves of interface wave. The solution is then guaranteed in terms of existence and uniqueness. As the propagation characteristics of interface waves are sensitive to shear wave parameters of solids only, some geoacoustic properties such as density and compressional parameters of media were then considered as "passive" parameters [12]. They were assumed to be constant during inversion. To estimate the shear wave attenuation of viscoelastic bottom, the shear wave velocity of the medium has to be identified at first (the shear wave velocity of solids can be well estimated from dispersion curves even if the attenuation is not determined with accuracy). Finally, a priori informations concerning the solution of the inverse problem, such as limiting physical values of the parameter, had to be introduced into the algorithm (see Figure 5). The estimation problem then reduces to the determination of the parameter $\boldsymbol{X}$ which satisfy equation (1) in the least-square sense and which then minimizes the cost function D:

Table II. Estimation of the shear wave velocity c_t of PVC and B2900 materials from experimental data issued from dispersion curves of the Stoneley-Scholte wave at the water/solid (PVC or B2900) interface for initial estimates of c_t: 0.75 c_t, 0.85 c_t, 0.95 c_t, 1.05 c_t, 1.15 c_t, 1.25 c_t. (In the PVC case the actual c_t is equal to 1100 m/s, and in the B2900 case the actual c_t is equal to 1620 m/s.)

PVC	estimate of c_t (m/s)	1057
	estimation errors	4%
B2900	estimate of c_t (m/s)	1547
	estimation errors	4.5%

$$D = \frac{1}{N} \sum_{k=1}^{N} \left(Y_{\mathrm{exp}k} - Y_{\mathrm{cal}k} \right)^2 . \tag{2}$$

A number of algorithms exist for solving this least squares minimization problem [13]. As there is only one parameter to be identified at a time, we chose the Brent's method which is a line search method. In fact, the method is a combination of the Golden Section Search and the inverse parabolic interpolation [13]. Note that in the case of multilayered media, i.e. when several parameters have to be recovered simultaneously, the Brent's method cannot be applied. In this case one can employ a neural network approach [14] for example.

Starting from an initial estimate for the shear wave parameter (velocity or attenuation), equation (2) was iteratively solved until the distance between the simulated data $Y_{\mathrm{cal}k}$ and the measured data $Y_{\mathrm{exp}k}$ came below some specified threshold value or until the difference between two consecutive updates of the parameter was negligible for all components of $\boldsymbol{X}$ (see Figure 5). Because of the possibility of reaching a local minimum of the cost function, the inversion had to be repeated with different initial estimates of the shear

Table III. Estimation of the shear wave attenuation α_t of PVC and B2900 materials from noisy data issued from energy curves of the Stoneley-Scholte wave at the water/solid (PVC or B2900) interface for different initial estimates of α_t. (In the PVC case the actual α_t is equal to 80 dB/m for 60 kHz, in the B2900 case the actual α_t is equal to 71 dB/m for 150 kHz.)

	Initial value of α_t (m/s)	0.75 α_t	0.85 α_t	0.95 α_t	1.05 α_t	1.15 α_t	1.25 α_t
PVC 60 kHz	estimate of α_t (dB/m)	91	83	83	86	90	90
	estimation errors	13%	4%	4%	7%	12%	13%
B2900 150 kHz	estimate of α_t (dB/m)	79	77	77	67	75	82
	estimation errors	11%	8%	9%	5%	6%	16%

Table IV. Estimation of the shear wave attenuation α_t of PVC and B2900 materials from experimental data issued from energy curves of the Stoneley-Scholte wave at the water/solid (PVC or B2900) interface for different initial estimates of α_t and for different number of input data for the inverse algorithm. (In the PVC case the actual α_t is equal to 88 dB/m for 75 kHz, in the B2900 case the actual α_t is equal to 57 dB/m for 70 kHz.) Initial value of α_t (dB/m): from 0.75 α_t to 1.25 α_t.

	Number of input data (exp. points)	(least-square)	Estimate of α_t (dB/m)	Estimation errors
PVC	8	-	88	0%
	-	8	88	0%
B2900	3	-	71	25%
	-	3	65	14%
	8	-	86	51%
	-	8	65	14%
	11	-	82	44%
	-	11	65	14%
	21	-	91	60%
	-	21	65	14%
	40	-	50	12%
	-	40	65	14%

wave parameter to determine if the solution was a true global minimum [15]. The input data for inversion algorithm were obtained by sampling either the dispersion or energy curves of interface waves at discrete points in the frequency space or in the propagation path space. A total of 8 data points were generally used in the inversion. The inversion was repeated 20 times for each initial estimate of the shear wave parameter and for each set of input data, in order to minimize erroneous solutions. The final estimate of the unknown parameter was then obtained by an average of the estimates for each inversion process.

3.2. Estimation of the shear wave velocity

The inverse procedure for determining the shear wave velocity of viscoelastic solids (PVC and B2900) modeling sediments was applied to dispersion curves of the Stoneley-Scholte wave. The actual shear wave velocity was equal to 1100 m/s for PVC material and 1620 m/s for B2900.

The inverse algorithm was first tested against noisy input data. These data were obtained by perturbing the theoretical data (see Figure A3) in a random way within a percentage of 10% from the exact values. The random sequence was obtained through the use of the function "Rand" (Matlab®) which generates uniform distribution of random numbers. Results are given in Table I for different initial estimates of the shear wave velocity c_t. Note that one has to be careful with the numerical solution of characteristic equation of interface wave by using different initial values of c_t which can be below or above the water sound speed. In particular, as the Stoneley-Scholte wave velocity is always below the slowest velocity of waves in media, one has to take a close look to the branch cuts, associated with the square root functions present in the characteristic equation (A10) (see Appendix), which have not to cross the region of the complex plane where the zeros of equation (A10) are seeked [9]. In Table I it can be seen that whatever the initial estimate for c_t, the shear wave velocity of materials is identified with a great accuracy. Note that the inversion error is always below 1% in the PVC case, and always below 6% in the B2900 case.

The inverse algorithm was also tested against experimental input data obtained from tank experiments [10, 11]. As no dispersion of the Stoneley-Scholte wave was noted, the input data for algorithm were constant and equal to 866 m/s for the PVC case and equal to 1260 m/s for the B2900 case (see Appendix). A total of 8 points were used in the inversion. Solving the inverse problem then implies to recover the shear wave velocity of viscoelastic media. Results are given in Table II for different initial estimates of the unknown parameter. Whatever the initial value of c_t, the shear wave velocities of PVC and B2900 are determined with accuracy. The estimation errors are below 5% in the two cases, which is in a good agreement with the inversion results from simulated data.

3.3. Estimation of the shear wave attenuation

The inverse procedure for determining the shear wave attenuation of viscoelastic solids (PVC and B2900) was applied to the energy curves of the Stoneley-Scholte wave. It has been shown in the previous section that the effect of the shear wave attenuation on the energy curves of the interface wave was the same whatever the frequency. Therefore only results obtained for a given frequency are presented here. The actual shear wave attenuation of PVC was equal to 80 dB/m for 60 kHz, while the attenuation of B2900 was equal to 71 dB/m for 150 kHz.

The inverse algorithm was tested against noisy input data. The input data were obtained by modifying the theoretical

data (see Figure A7) in a random way within a percentage of 50% from the actual values. Results are given in Table III for different initial estimates of the shear wave attenuation α_t. It can be seen that although the estimation errors are quite great (below 13% for the PVC case and below 16% for the B2900 case), the identification of the shear wave attenuation of materials is accurate enough for a rough identification of viscoelastic solid properties.

The inverse algorithm was also tested against experimental data obtained from tank experiments. The input data for inversion algorithm were selected from points of the experimental energy curves of the Stoneley-Scholte wave (see Figure A7). The inversion gives the shear wave attenuation of viscoelastic media for a given frequency. The actual shear wave attenuation was 88 dB/m for 75 kHz for PVC and 57 dB/m for 70 kHz for B2900. Results are given in Table IV for different initial estimates for the unknown parameter and for different numbers of experimental points used as input data. Whatever the value of the initial estimate, the shear wave attenuation of PVC material is identified with a high accuracy, owing to the high quality of the experimental results obtained from tank experiments. The estimation errors are close to 0%. On the contrary, the shear wave attenuation of B2900 material is not recovered with accuracy whatever the number of input data: the inversion errors are thus close to 50%. This discrepancy comes essentially from the fact that in the B2900 case each experimental point can be very different from the theoretical one, although experimental data are globally close to the theoretical curves. Has the difference in the shear wave velocity of the two materials (PVC has S-wave velocity less than water sound speed, B2900 higher) anything to do with it? Such a conclusion cannot be drawn as long as experiments with another material with S-wave velocity higher than water sound speed are not performed. In the B2900 case, the lack of correlation between experimental data implies bad estimation of the shear wave attenuation of the viscoelastic solid. The discrepancy can be overcome by increasing the number of input data of the inversion algorithm: using 11 points (respectively, 40 points) in the inversion leads to an estimation error close to 44% (respectively, 12%) (see Table IV). But increasing the number of input data of the inversion algorithm can sometimes imply convergence problems of algorithms. Therefore the better way to solve the inverse problem when the material properties are not known is to select points from the least-square line which fits the experimental data. The correlation obtained from measured data thus implies a quite good estimation of the shear wave attenuation of the viscoelastic material. In the B2900 case, the estimation errors are then close to 14% whatever the initial estimate of α_t and whatever the number of input data. These results are thus in a good agreement with those obtained from simulated data (see Table III). In the PVC case, the shear wave attenuation of material is perfectly identified, owing to the very good agreement between experimental results obtained from tank experiments and theoretical predictions. In particular, the least-square line which fits the experimental data is identical to the theoretical energy curve of the Stoneley-Scholte wave. The Brent's method has been well suited for our purpose of demonstrating the possibility of recovering the shear wave parameters of viscoelastic solids from dispersion or energy curve data. However the estimation generally depends on the initial estimates of the unknown parameters. A better way to solve the inverse problem would be to use the neural network approach which would be suitable for the determination of both the shear wave velocity and the shear wave attenuation in solids. No initial estimate of the unknown parameter close to the exact one and no a priori knowledge of the environment are required [14, 16]. Nevertheless the study presented in this section has proved the high interest in modeling underwater acoustics problems by tank experiments on well-controlled media in laboratory. It can be considered as a first step towards acceptable identification of geoacoustical properties of viscoelastic media.

4. Conclusion

Many other previous works have shown that it is possible to determine the geoacoustical properties of sediments by remote techniques. Interface waves, such as Stoneley-Scholte waves, observed over specific propagation paths at the water/sediment interfaces may contain sufficient information to deduce the properties of solids and in particular the shear wave parameters of sedimentary bottom. But how to be sure of the accuracy of the estimated parameters of non-controlled media? The aim of the paper was then to give an answer to this question and to show that testing the performance of inversion methods on synthetic materials which can model marine sediments might be a very useful tool and might then be applied to the Underwater Acoustics field. It could be considered as a first step before the final ground thruth performed at sea under realistic conditions. In the study we considered the possibility of recovering the parameters of well-controlled media with the propagation characteristics of Stoneley-Scholte waves at water/absorbing solid interfaces. In particular we used in inversion studies experimental data obtained from tank experiments. It has been shown that the shear wave velocity of solids could be reliably identified from dispersion curves of interface waves. A strategy for determining the shear wave attenuation of solids has been discussed when the material properties are not known. The estimation of the shear wave attenuation from the energy curves of interface waves, expressing the amount of energy lost along a propagation path, was then accurate. But the estimation quality seems to depend on the solid materials. The conclusion that the difference in the shear wave velocity of the two materials (PVC has S-wave velocity less than water sound speed, B2900 higher) has maybe something to do with it but it cannot however be drawn so long as experiments with another material with S-wave velocity higher than water sound speed are not performed.

Our method has been developed under the assumption of viscoelastic media. In future works, and in particular for people interested in the study of marine sediments, it would be necessary to modify our theory and experiments in order to consider poroviscoelastic or inhomogeneous viscoelastic media.

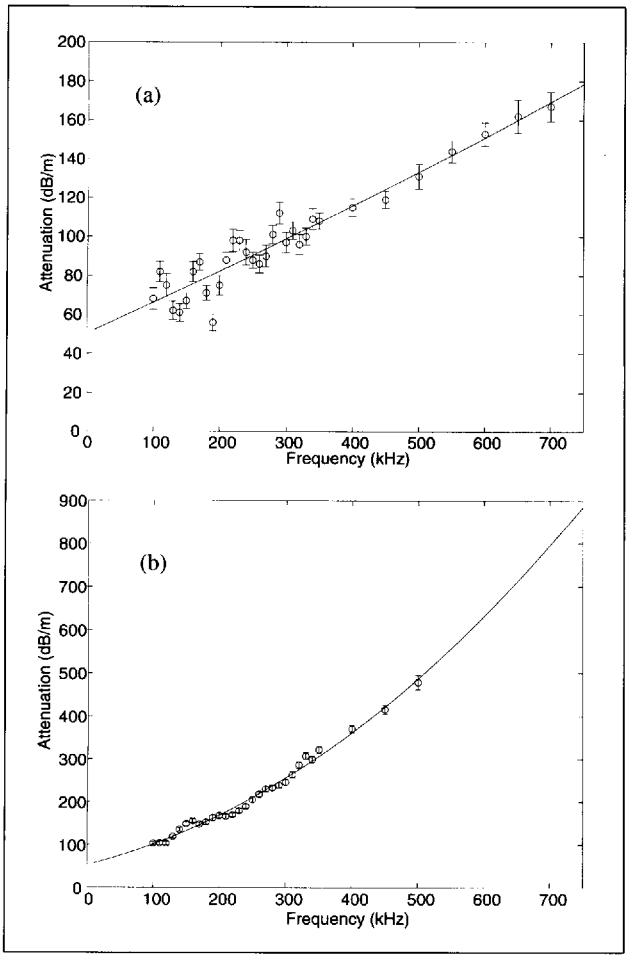

Figure A1. Measured attenuations of the plane waves in PVC material. (a) longitudinal waves; (b) shear waves. o: experimental values, ——: the slope obtained by the least-squares fit of the experimental data.; I: error ranges.

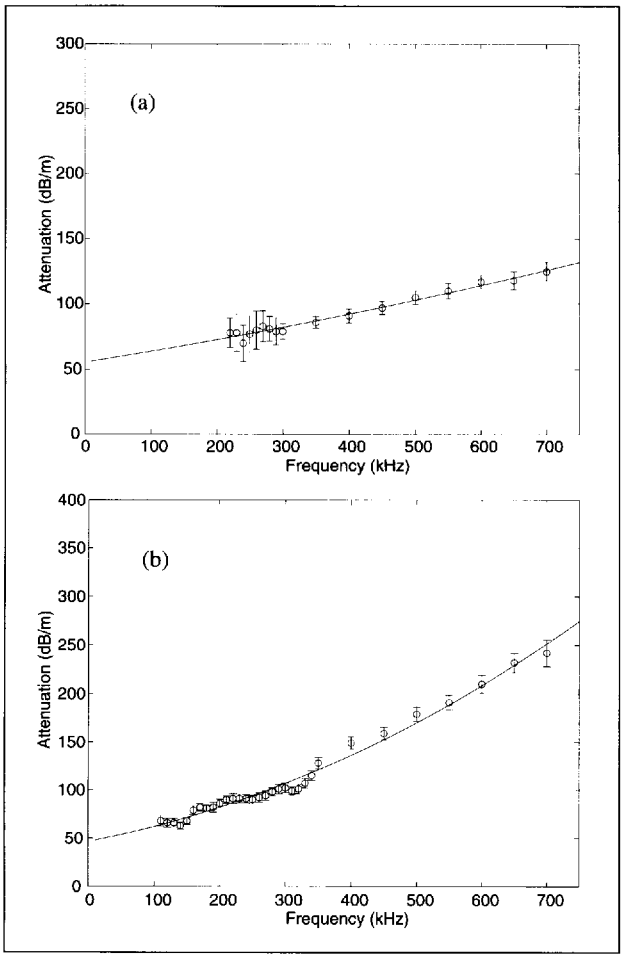

Figure A2. Measured attenuations of the plane waves in B2900 material. (a) longitudinal waves; (b) shear waves. o: experimental values, ——: the slope obtained by the least-squares fit of the experimental data; I: error ranges.

Appendix

The propagation of the Stoneley-Scholte wave at the ideal fluid/viscoelastic solid interface has been studied in previous papers [9, 10, 11]. Only a synthetic summary is presented in this section. Since the analysis of the propagation of interface wave was developed by using the approach of inhomogeneous plane waves, it seems necessary to recall first some properties of the waves in viscoelastic [17] media and in fluid media [18].

A1. Review of some properties of inhomogeneous plane waves

A1.1. In a viscoelastic solid medium

Two inhomogeneous plane waves can propagate into a solid medium, assumed to be viscoelastic, homogeneous, isotropic and infinite: a lamellar one ($\mathbf{curl}\boldsymbol{u}_L = \mathbf{0}$) and a torsional one ($\mathrm{div}\boldsymbol{u}_T = 0$). The acoustic real-valued displacements $\boldsymbol{u}_L$ and $\boldsymbol{u}_T$ associated with these waves are sought in the general form:

$$\boldsymbol{u}_M = \Re e\{\boldsymbol{u}_M^*\}, \tag{A1}$$

with:

$$\boldsymbol{u}_M^* = \boldsymbol{u}_M^{0*} \exp(\mathrm{i}\boldsymbol{K}_M^* \cdot \boldsymbol{r}), \tag{A2}$$

$$\boldsymbol{K}_M^* = \boldsymbol{K}_M' + \mathrm{i}\boldsymbol{K}_M'', \tag{A3}$$

$$\boldsymbol{K}_M^* \cdot \boldsymbol{K}_M^* = k_M^*(1 + \mathrm{i}\omega\nu_m). \tag{A4}$$

The subscript $M = L, T$ is used to denote the lamellar and torsional waves. $\boldsymbol{u}_M^{0*}$ is a complex-valued constant vector and $\boldsymbol{K}_M^*$ the complex-valued wave vector of the inhomogeneous waves. The real-valued vectors $\boldsymbol{K}_M'$ and $\boldsymbol{K}_M''$, respectively, represent the propagation and the attenuation of the wave. k_m are the wave numbers of the associated homogeneous plane waves (longitudinal ($m \sim l$) and shear ($m \sim t$) ones). The quantities ν_m are defined by the ratio between the viscosity coefficients (Λ' and M') and the Lamé coefficients (Λ and M):

$$\nu_{(m\sim l)}(\omega) = \frac{2M' + \Lambda'}{\rho c_l^2}, \tag{A5}$$

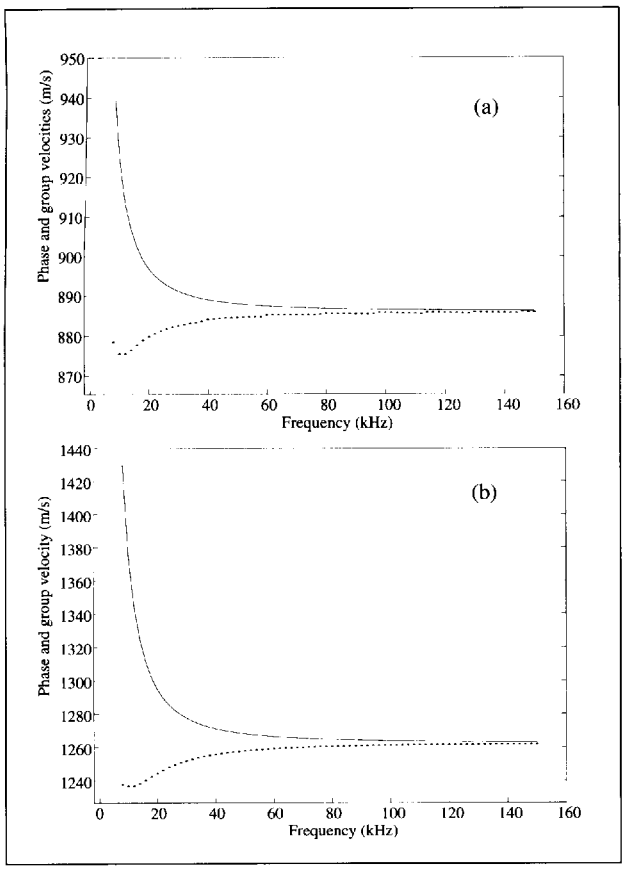

Figure A3. Theoretical dispersion curves of the phase (——) and group (· · ·) velocities of the Stoneley-Scholte wave. (a) For the water/PVC interface; (b) for the water/B2900 interface.

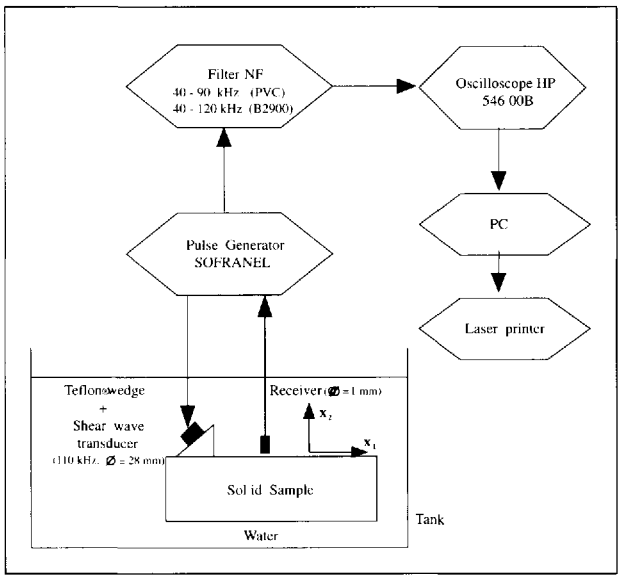

Figure A4. Schematic representation of the experimental set up.

$$\nu_{(m\sim t)}(\omega) = \frac{M'}{\rho c_t^2}, \tag{A6}$$

ρ being the density of the solid and $c_{(m\sim l)} = \sqrt{(2M+\Lambda)/\rho}$ (respectively $c_{(m\sim t)} = \sqrt{M/\rho}$) the velocity of the homogeneous longitudinal (respectively shear) plane waves. $\boldsymbol{r}$ is the space vector and ω is the angular frequency.

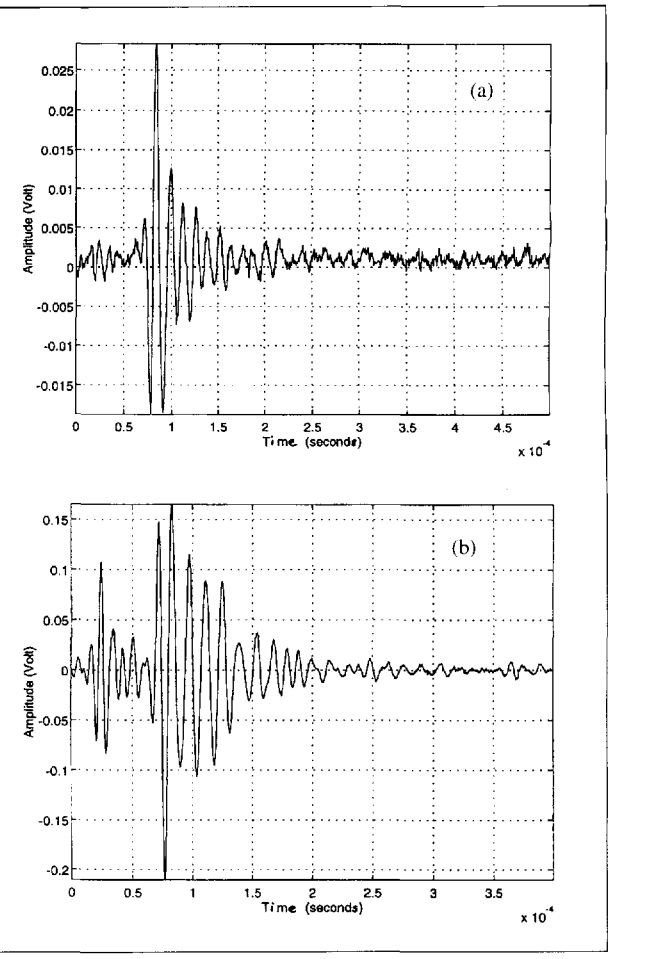

Figure A5. Signals detected by the receiver, (a) at the water/PVC interface, wedge-receiver distance = 6 cm; (b) at the water/B2900 interface, wedge-receiver distance = 3.8 cm.

The quantity ν_l (respectively, ν_t) can be expressed in terms of the measured attenuation of the longitudinal homogeneous (respectively, "transversal" inhomogeneous) plane waves in the solid:

$$\nu_l(\omega) = \frac{2\alpha_l(\omega)}{\omega k_l(\omega)}, \tag{A7}$$

$$\nu_t(\omega) = \frac{2\alpha_t(\omega)\cos\Theta_r}{\omega k_t(\omega)}. \tag{A8}$$

Θ_r is the refracted wave angle in the solid material [11].

Equation (A4) implies that the vectors representing propagation and damping of the inhomogeneous plane waves are not perpendicular: in this case the waves are called heterogeneous waves.

A1.2. In an ideal fluid medium

Only one inhomogeneous plane wave can propagate into an ideal fluid: a lamellar one ($\mathbf{curl}\boldsymbol{u}_F = \mathbf{0}$). The general form of the real-valued displacement $\boldsymbol{u}_F$ associated with this

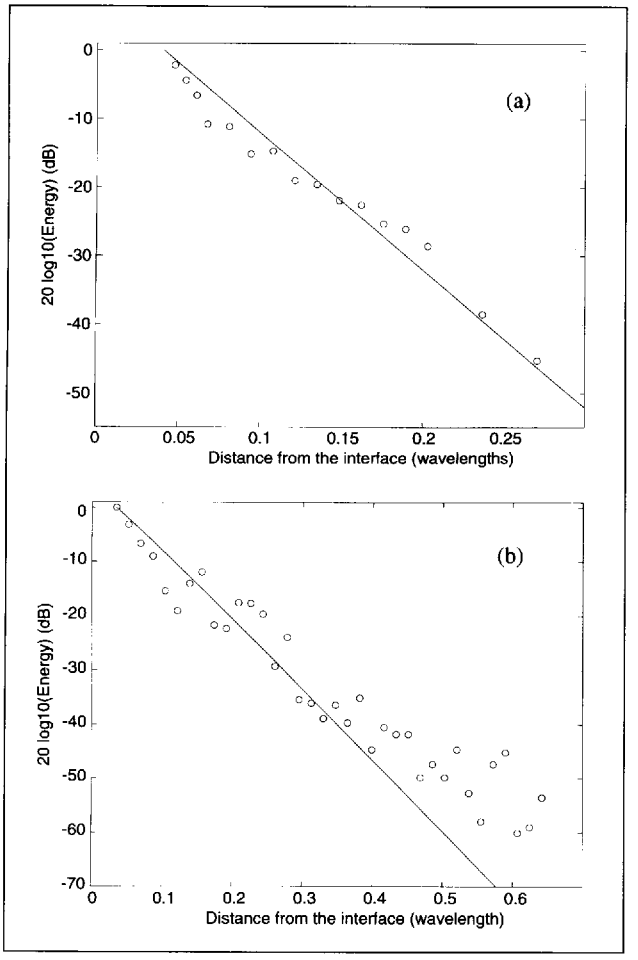

Figure A6. Differences between theoretical (——) and experimental (o) results quantified in dB: attenuation of the energy of the Stoneley-Scholte wave as a function of distance: (a) from the water/PVC interface, frequency 60 kHz, wavelength λ = 1.5 cm, normalization of energy with the energy obtained for the first position of the receiver (0.6 mm from the interface and 2 cm from the source); (b) from the water/B2900 interface, frequency 110 kHz, wavelength λ = 1.2 cm, normalization of energy with the energy obtained for the first position of the receiver (0.4 mm from the interface and 3 cm from the source).

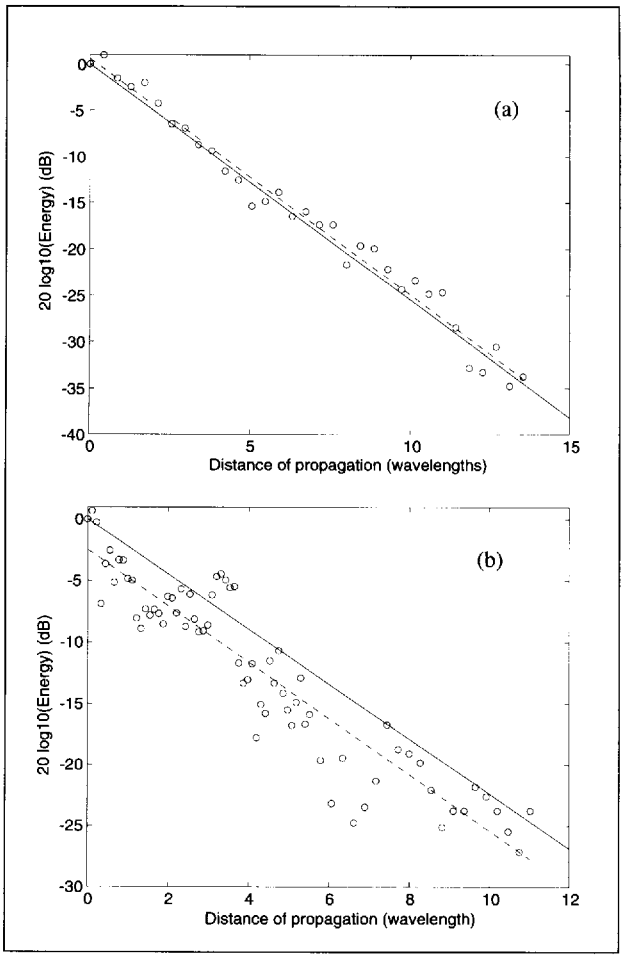

Figure A7. A comparison between theoretical (——) and experimental (o) results: attenuation of the energy of the Stoneley-Scholte wave as a function of distance of propagation at the interface. (a) The water/PVC interface, frequency 75 kHz, wavelength λ = 1.2 cm; (b) the water/B2900 interface, frequency 70 kHz, wavelength λ = 1.8 cm. Normalization of energy with the energy obtained for the first position of the receiver (3 cm from the source). Differences between theoretical and experimental data quantified in dB. – – –: The slope obtained by the least squares fit of the experimental data.

wave is given by equations (A1–A3). Since the medium is not attenuating, equation (A4) then becomes:

$$K_F^* \cdot K_F^* = k_0^2. \tag{A9}$$

$k_0 = \omega/c_0$ is the wave number of the associated homogeneous wave, $c_0 = \Lambda_0/\rho_0$ the longitudinal wave velocity, ρ_0 the fluid density and Λ_0 the Lamé coefficient. Equation (A9) implies that the vectors representing propagation and damping of the inhomogeneous plane wave are perpendicular: in this case the wave is called evanescent wave.

A2. The Stoneley-Scholte wave at the plane boundary between an ideal fluid and a viscoelastic solid

The propagation of the Stoneley-Scholte wave has been theoretically analyzed at the plane boundary between an ideal fluid and a viscoelastic solid [9]. The two media were supposed to be homogeneous, isotropic and semi-infinite. The interface wave was locally sought in the form of a linear combination of three inhomogeneous waves (one evanescent wave in the fluid and two heterogeneous waves in the solid), which satisfy boundary conditions at the interface [10].

A2.1. Theoretical background

One now introduces the perpendicular axes $0x_1$ and $0x_2$ with unit vectors $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ such that the interface lies along $0x_1$, the fluid medium being defined by $x_2 > 0$ and the solid one by $x_2 < 0$.

The displacement of the lamellar evanescent wave in the fluid and those of the lamellar and torsional heterogeneous waves in the viscoelastic solid are described in Section A.1. All these waves have the same projection $\boldsymbol{K}_1^*$ of their wave vectors upon the axis $\boldsymbol{x}_1$. The required conditions of con-

Table A1. Some measured characteristics of the fluid (water) and the solids (B2900 and PVC).

Frequencies 10 kHz–1 MHz	Density (kg/m^3)	LW velocity (m/s)	SW velocity (m/s)
Water	1000	1478	–
PVC	1360	2270	1100
B2900	2000	2910	1620

tinuity at the interface ($x_2 = 0$), that is the slip boundary condition, give the characteristic equation of the Stoneley-Scholte wave in the unknown K_1^*. This may be put in the following form [9, 10]:

$$\begin{aligned}&(K_1^{*2} - K_{T2}^{*2})W + 2K_1^{*2}K_{L2}^*K_{T2}^*X \\ &= \frac{\rho_0}{\rho}\frac{K_{L2}^*}{K_{F2}^*}k_t^4(1 - \mathrm{i}\omega\nu_t),\end{aligned} \tag{A10}$$

with:

$$K_{L2}^* = \pm\sqrt{k_l^2(1 + \mathrm{i}\omega\nu_l) - K_1^{*2}}, \tag{A11}$$

$$K_{T2}^* = \pm\sqrt{k_t^2(1 + \mathrm{i}\omega\nu_t) - K_1^{*2}}, \tag{A12}$$

$$K_{F2}^* = \pm\sqrt{k_0^2 - K_1^{*2}}, \tag{A13}$$

$$\begin{aligned}W &= 2K_1^{*2}(1 - \mathrm{i}\omega\nu_\theta) + \mathrm{i}\omega(c_l^2/c_t^2) \\ &\quad \cdot (\nu_l K_{L2}^{*2} + \nu_\theta K_1^{*2}) - Y,\end{aligned} \tag{A14}$$

$$Y = k_t^2(1 + \mathrm{i}\omega\nu_l), \tag{A15}$$

$$X = 2(1 - \mathrm{i}\omega\nu_\theta) + \mathrm{i}\omega(c_l^2/c_t^2)(\nu_\theta - \nu_l), \tag{A16}$$

$$\nu_\theta = 2\frac{c_l^2\nu_l - 2c_t^2\nu_t}{c_l^2 - 2c_t^2}. \tag{A17}$$

The terms K_{M1}^* and K_{M2}^* are the projections of the vector $\boldsymbol{K}_M^*$ upon the axes $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$. The solution of equation (A10) in the complex plane K_1^* gives the Stoneley-Scholte wave velocity and attenuation for a given frequency. According to the finite energy principle in both media, the branches for the different square root functions are selected to be such that:

$$\Im m\{K_{L2}^*, K_{T2}^*\} < 0, \tag{A18}$$

$$\Im m\{K_{F2}^*\} > 0. \tag{A19}$$

The densities of the energy in the fluid and in the solid, associated with the Stoneley-Scholte wave and expressed in terms of the components of the strain tensor and its complex-conjugate [10], decrease exponentially from both sides of the interface. Damped propagation takes place along the interface with a speed c_{sch} given by:

$$C_{sch} = \omega/K_1', \tag{A20}$$

$$K_1' = \Re e\{K_1^*\} > \max(k_0, k_t, k_l). \tag{A21}$$

A2.2. Numerical calculations

The ideal fluid was represented by water. To simulate a viscoelastic bottom we used either a material called B2900 (it is a synthetic resin which can be loaded with alumina and tungsten and whose velocities are such that $c_l > c_t > c_0$), or a PVC material (with $c_t < c_0 < c_l$). The acoustical characteristics of the media, assumed to be constant in the frequency range 10 kHz–1 MHz, are summarized in Table A1. The attenuations of the plane waves in the solid increase with frequency (see Figures A1 and A2). For 500 kHz, the longitudinal wave attenuation was 110 dB/m for B2900 and 130 dB/m for PVC, and the "shear" wave attenuation was 180 dB/m for B2900 and 490 dB/m for PVC.

The numerical solution of equation (A10) for each frequency provided the dispersion curve of the phase velocity of the Stoneley-Scholte wave (see Figure A3). The dispersion, due to the absorbing property of the solid, is weak and it takes place in only a small frequency range (10–50 kHz). The group velocity is obtained by numerical differentiation of the phase velocity with respect to frequency [19]:

$$c_g(f) = c_\phi(f) + \frac{f\dfrac{\partial c_\phi(f)}{\partial f}}{1 - \dfrac{f}{c_\phi(f)}\dfrac{\partial c_\phi(f)}{\partial f}}. \tag{A22}$$

c_ϕ is the phase velocity and c_g is the group velocity. They both depend on the frequency f.

Study of the distribution of the acoustical energy in both media (fluid and solid) as a function of the distance from the interface showed that 58% of energy associated with the Stoneley-Scholte wave is concentrated in the fluid [10]. The energy does not penetrate deep into the two media (penetration length less than one Stoneley-Scholte wavelength). Moreover, as the solids are absorbing, energy of the interface wave is quickly attenuated during its propagation at the boundary between water and the viscoelastic solids. The latter remark reveals the difficulty in detecting the Stoneley-Scholte wave at the water/viscoleastic solid interface. However, as the amount of energy lying in the viscoelastic solid is great, one can recover the properties of the solid. This assumption will be confirmed subsequently.

A2.3. Experimental verification of numerical results

The purpose of this subsection is to validate the theoretical results obtained for the Stoneley-Scholte wave properties by tank experiments. In particular, it was desired to determine quantitatively the energy damping of the interface wave as a function of propagation path along the boundary between water and viscoelastic solids.

The Stoneley-Scholte wave was generated by a solid wedge excited by a shear wave transducer (diameter 28 mm, resonance frequency 110 kHz) [10]. The wedge was made of Teflon®, a plastic material with weak shear wave velocity (measured velocity: 550 m/s). The angle of the wedge was calculated to ensure the conversion of an incident shear wave into an interface wave at the water/ solid material (PVC or B2900) interface. The transmitter and the solid wedge were bonded by a coupling, respectively to the Teflon® material and to the solid sample (PVC or B2900). The signals were received by a wideband hydrophone (diameter 1 mm) situated

in the water near the surface of the solid sample (distance less than 1 mm). The experiments took place in a tank of average dimensions ($2.5 \times 1 \times 1$ m^3) fitted out with stepping motors. The parallelepipedic samples were immersed in pure water to a depth 300 mm. The thicknesses of the samples (PVC: 83 mm, B2900: 78 mm) were large relative to the wavelengths of the Stoneley-Scholte wave ($\lambda_{s,\mathrm{PVC}} = 8$ mm, $\lambda_{s,\mathrm{B2900}} = 1.2$ cm at the frequency 110 kHz). In both experiments, wideband pulsed waves were produced by a pulse generator and the received signals were low-pass filtered in order to increase the signal to noise ratio (see Figure A4).

Typical waveforms are represented on Figure A5. It was concluded that the most significant signal detected near the surface of the solid sample (PVC or B2900) by the receiver corresponds to the Stoneley-Scholte wave excited by the shear component of the emitter, while the first signal corresponds to a signal issued from the longitudinal component of the emitter. These facts were entirely verified by measuring the velocity of the presumed Stoneley-Scholte wave. The measured value of the velocity c_0 of sound in water was 1478 m/s. The experimental measurement of the velocity c_{sch} of the Stoneley-Scholte wave was on average 866 ± 10 m/s for the water/PVC interface and 1260 ± 10 m/s for the water/B2900 interface. The difference between theoretical and experimental velocities of the Stoneley-Scholte wave was below 3%. No dispersion of the interface wave was unfortunately observed.

The higher the distance from the water/PVC or water/B2900 interface, the higher the decrease of the energy of the Stoneley-Scholte wave. The differences between the theoretical values and the experimental ones are reported in a logarithmic scale in Figure A6. The curves are normalized to the energy of the signal observed for the first position of the receiver. The initial distance of the receiver from the wedge was 2 cm (for PVC) or 3 cm (for B2900) and the initial distance of the receiver from the interface was 0.6 mm (for PVC) or 0.4 mm (for B2900). For each experiment, the agreement between the measured data and the theoretical prediction was good for the whole frequency spectrum. The energy of the signal associated with the Stoneley-Scholte wave also decreased exponentially during the wave propagation and the interface wave was completely attenuated at the boundary between water and the solid samples at the end of a 15-wavelength propagation, as predicted by theory (see Figure A7). These findings thus prove the validity of the theoretical study [9] of the Stoneley-Scholte wave at the plane boundary between an ideal fluid and a viscoelastic solid. They are of great interest for study of inverse problem and they are used as data for inversion algorithm, as it is shown in Section 3.

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