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ON THE INFLUENCE OF THE INTERFACE FRESNEL ZONE FOR ESTIMATING MEDIA PARAMETERS FROM SEISMIC AMPLITUDE-VERSUS-ANGLE CURVES

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Abstract

The band-limited property and the low-frequency content of the seismic data are rarely taken into account in wave reflection modeling. In this work we point out the consequences of ignoring such signal characteristics in both forward and inverse modeling of seismic wave reflection. The variation in the reflected P-wave amplitude as a function of the incidence angle, also known as AVA curve, obtained with the classical plane-wave (PW) theory, is compared with the exact solution provided by the 3D code OASES, and with our approximation which accounts for the spatial region which physically contributes to the wave reflection process, i.e. the Interface Fresnel zone. Our approximation provides much better data predictions than the PW theory, widely used in AVA studies in seismic exploration. Moreover, assuming that the AVA curves corresponding to real data may be well described by the PW theory leads to inaccurate estimations of the media properties. Our approximation may be therefore an attractive alternative to classical methods to extract information relative to bottom parameters from AVA signals.

Introduction

In seismic reflection surveys the waves generated by a point source propagate in the stratified Earth, and are recorded at the surface by the receivers, after being reflected by the interfaces. Classically in geophysical exploration, the so-called technique AVO (Amplitude Variation with Offset), respectively AVA (Amplitude Variation with Angle), uses the variability of the reflected P-wave amplitude with source-receiver distance, respectively with incidence angle, to constrain the reflector location and the media properties. Since the media heterogeneity can be highly complex in the seismic frequency range (typically, between 10 and 60 Hz), retrieving these characteristics is actually a difficult task. Solving such an inverse problem implies that one could find a set of media parameters which may fit the propagation measurements, and therefore that a theoretical model of wave propagation was first developed. A survey of the literature brings to light that most calculations are generally performed within the framework of monochromatic plane-wave (PW) theory [3]. The amplitude of the reflected P-waves is thus evaluated knowing the characteristics of the source and the PW reflection coefficient

at the interface, and considering the geometrical-spreading compensation. The angular dependence of reflection coefficients for plane waves impinging on a plane interface between two semi-infinite, homogeneous, isotropic, and elastic media is exactly described by the Zoeppritz equations [1]. Nevertheless, as the complexity of these equations defies physical insight and prevents from processing simple inversion techniques for estimating parameters of real media, useful linearized versions of the exact plane wave reflection coefficients [1] have been therefore commonly applied in AVO/AVA analyses [3]. These approximations, valid only for typical range of small incidence angles and for weak impedance contrast between media, have greatly facilitated physical understanding and parameter estimation, but are nowadays becoming obsolete with the computer power. The underlying assumptions of the PW theory, illustrated by the Zoeppritz equations, are the infinite frequency and the infinite bandwidth of the signals. However, the measured seismic data are band-limited signals and have a low frequency content. A question then arises: what is the effect of these signal characteristics on wave reflection modeling and on estimation of media properties? In the paper we focus on this question.

Besides the PW theory, the basis of many seismic studies is the ray theory. Under this approximation it is assumed that the high-frequency part of elastic energy propagates along infinitely narrow lines through space, called rays, which join the source and the receiver. Ray theory is then strictly valid only in the limit of a hypothetical infinite-frequency wave. As measured seismic data have a low frequency content, it is accepted that seismic wave propagation is extended to a finite volume of space around the ray path, called the 1st Fresnel volume [10], which contributes to the received wavefield for each frequency. The 1st Fresnel volume, hereafter denoted FV, and its intersection with a reflector, called the Interface Fresnel zone (IFZ), have received broad attention in recent past years. These concepts are continually being developed and have found so many applications in seismology and in seismic exploration that it is impossible here to review all the books and articles which pay attention to them in seismic wave propagation [14, 15, 17]. Nevertheless, we shall mention the works of Červený and his co-authors who have suggested two methods for including FV parameter calculations into the ray tracing procedure in complex 2D and 3D structures. The first one, called the Fresnel volume ray tracing, combines the paraxial ray approximation with the dynamic ray tracing, and is only applicable to zero-order waves (direct, reflected and transmitted waves...), whereas the second method, more accurate than the previous one, is based on network ray tracing [16]. They have also derived analytical expressions for FVs of seismic body waves and for IFZ for simple structures, which offers a deeper insight into the properties of FV and IFZ [11, 12]. Of particular interest are the size of the IFZ and the size of the volume of the reflector involved in reflection time measurements [7] because each one can be related to the horizontal and vertical resolutions of seismic methods [13]. Unfortunately, as Červený and co-authors' objectives were concerned essentially with kinematic ray tracing, the expressions they derived are incomplete. Until now, only the IFZ and the penetration depth of the FV below the reflector were considered in studies. Nevertheless, if the seismic amplitudes at receivers have to be evaluated, we must determine the interface reflectivity by accounting for the whole spatial region in the vicinity of the interface which affects it. In other words, we must account for the IFZ and for certain volumes below the interface in the transmission medium and above the interface in the incidence medium which define the reflector from the seismic viewpoint [5]. If there is no heterogeneity in the vicinity of the interface, only the IFZ must be considered for the computation of the interface reflectivity. On the other hand, if only traveltime measurements are considered, for instance, for locating the reflectors in the media, there is no need for defining the region above the interface, because this region is already included in the representation of the FV. In this case, only the region beyond the interface has to be considered. The consequences of ignor-

ing the IFZ in forward modeling of wave reflection process have been recently pointed out in [4], however, to the best of the authors' knowledge, the role of the IFZ in the inverse modeling has not been investigated yet. This is precisely the goal of our work. To focus specifically on the imprint of the IFZ, we consider a very simple elastic model, e.g. a smooth homogeneous interface between homogeneous, isotropic, and elastic media.

The paper is divided in two sections. Section 1 briefly recalls the concept of FV and IFZ. Section 2 investigates the role of the IFZ in both forward and inverse modeling of wave reflection. The variation in the reflected P-wave amplitude as a function of the incidence angle, evaluated with our approximation which combines the IFZ concept with the Angular Spectrum Approach (ASA) [6], is compared with the classical PW reflection coefficient, and with the exact solution obtained with the 3D code OASES (http://acoustics.mit.edu/faculty/henrik/oases.html). The influence of the classical PW theory framework which does not account for the IFZ, widely used for AVO/AVA studies, on the estimation of elastic media properties is also investigated by simple inversion processing. Finally, we discuss the implications of our approximation in the estimation of media properties.

1 General background: the concept of Fresnel volume and Interface Fresnel zone

We consider two homogeneous isotropic elastic media in welded contact at a plane interface located at a distance z_M from the xy-plane including the point source S $(-x_S,0,0)$ and the receiver R $(x_S,0,0)$. We assume that the interface is isolated from the other ones. We mean that the distance between this interface and another one is much greater than $\frac{V}{2B}$, where V is the medium velocity and B is the frequency bandwidth of the source. The source generates in the upper medium a spherical wave with a constant amplitude. The spherical wave can be decomposed into an infinite sum of PW synchronous each other at the time origin. We consider the harmonic PW with frequency f which propagates in the upper medium with the velocity V_{P1} from S to R, after being reflected by the interface at the point M$(0,0,z_M)$ in a specular direction θ with respect to the normal to the interface (Fig. 1). Let the traveltime of the specular reflected wave be t_{SMR}.

The set of all possible rays SM_iR with constant traveltime t_{SMR} defines the isochrone for the source-receiver pair (S,R) relative to the specular reflection SMR. This isochrone describes an ellipsoid of revolution tangent to the interface at M, whose rotational axis passes through S and R. By definition, the FV corresponding to S and R and associated with the reflection at M is formed by virtual diffraction points F such that the waves passing through these points interfere constructively with the specular reflected wave. This condition is fulfilled when the path-length difference is less than one-half of the wavelength $\lambda_1 = \frac{V_{P1}}{f}$ corresponding to the dominant frequency f of the narrow-band source signal [10]:

$$|l(F,S)+l(F,R)-l(M,S)-l(M,R)| \leq \frac{\lambda_1}{2} \quad , \tag{1}$$

the quantity $l(X,Y)$ denoting the distance between the point X and the point Y. As is well-known, the main contribution to the wavefield comes from the 1st FV as the rapid oscillatory responses of the higher-order FVs and Fresnel zones cancel out and give minor contributions to the wavefield [2]. In our work we restrict ourselves to the 1st FV which is simply referred to as FV. The FV is represented by the volume situated above the interface in the upper medium and bounded by two ellipsoids of revolution, with foci at S and R, tangent to fictitious parallel

planes to the interface and located at a distance $\frac{\lambda_1}{4}$ below and above the interface (Fig. 1). The two ellipsoids of revolution are defined by:

$$\frac{x^2}{\left(\frac{z_M}{\cos\theta} \pm \frac{\lambda_1}{4}\right)^2} + \frac{y^2}{\left(\frac{z_M}{\cos\theta} \pm \frac{\lambda_1}{4}\right)^2 - z_M^2 \tan^2\theta} + \frac{z^2}{\left(\frac{z_M}{\cos\theta} \pm \frac{\lambda_1}{4}\right)^2 - z_M^2 \tan^2\theta} - 1 = 0 \quad . \qquad (2)$$

Note that, as seismic wavefields are transient and large-band, it is generally necessary to decompose the source signal into narrow-band signals for which monochromatic FV can be constructed for the prevailing frequency of the signal spectrum [9].

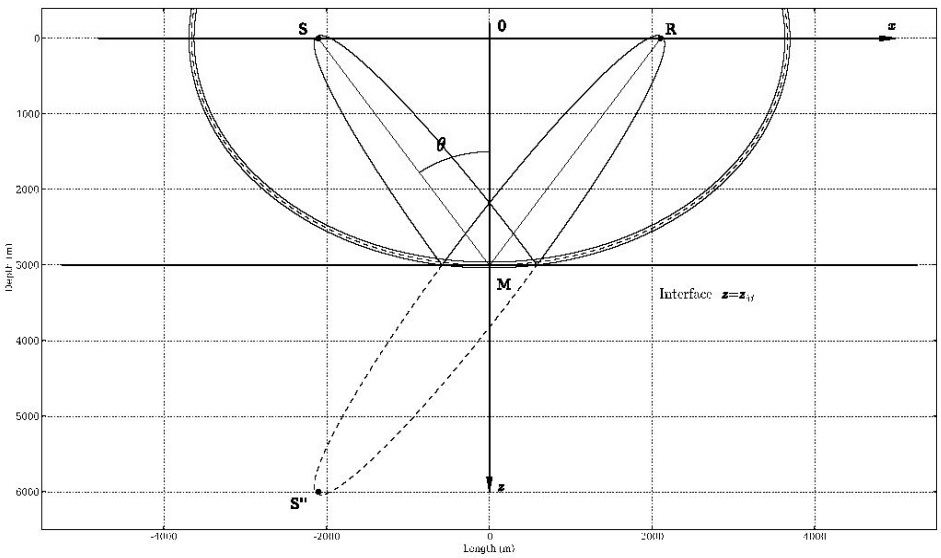

Fig. 1: Representations, in the xz-plane, of the Fresnel volume involved in the wave reflection at the point M at a plane interface, under the incidence angle $\theta = 35°$. The source S and the receiver R are situated at a distance 3000 m from the interface. The classical representation of the Fresnel volume is the ellipsoid of revolution with foci located at R and at the image source S". Another representation of the Fresnel volume associated with the reflection SMR is given by the volume located in the incidence medium between the ellipsoids of revolution with foci at S and R (see the text for more details). The velocities in the upper and lower media are respectively $V_{P1} = 4000$ m/s and $V_{P2} = 5200$ m/s, and the frequency $f = 32$ Hz. The seismic wavelengths in the upper and lower media are respectively $\lambda_1 = 125$ m and $\lambda_2 = 162.5$ m. The critical angle is equal to $\theta_C = 50.28°$.

The IFZ is defined as the cross section of the FV by an interface which may not be perpendicular to the ray SM. If the source S and the receiver R are situated at the same distance from the interface, the IFZ is represented by an ellipse centered at the reflection point M, whose equation is obtained from the formulation of the ellipsoid of revolution, eq.2, keeping the sign + and replacing z by z_M. The in-plane semi-axis $r^{\parallel}$ and the transverse semi-axis $r^{\perp}$ of the IFZ are then expressed as [11]:

$$r^{\parallel} = r^{\perp}\left[1 - \frac{z_M^2 \tan^2\theta}{\left(\frac{z_M}{\cos\theta} + \frac{\lambda_1}{4}\right)^2}\right]^{-\frac{1}{2}} \quad , \; r^{\perp} = \left[\frac{\lambda_1}{2}\left(\frac{z_M}{\cos\theta} + \frac{\lambda_1}{8}\right)\right]^{\frac{1}{2}} \quad . \qquad (3)$$

The characteristics of the IFZ depend on the positions of the source-receiver pair, and also on the incidence angle of the ray SM. Moreover, larger portions of the interface are involved for low-frequency than for high-frequency components of the wavefield, and also for great incidence angles θ rather than for small angles (Fig. 2). It is also well-known that a perturbation of the medium actually affects the reflected wave when this perturbation is located inside the IFZ.

As an aside, we should point out here that in many papers is used the classical representation of the FV which is an ellipsoid of revolution with foci located at R and at the mirror image S" of the source S (Fig. 1). This representation, mainly based on transmission considerations, is suitable for accounting for the heterogeneities of the medium body located in the vicinity of the ray, while the FV representation we use is more appropriate to account for the heterogeneities of the interface, as it is connected strictly to the wave reflection process. Moreover, unlike the classical one, this representation allows the definition of the volumes above and beyond the interface which characterize the reflector [5]. Note that the two representations are complementary and must be combined if the wave propagation in media with heterogeneities in the body and at the interfaces is investigated.

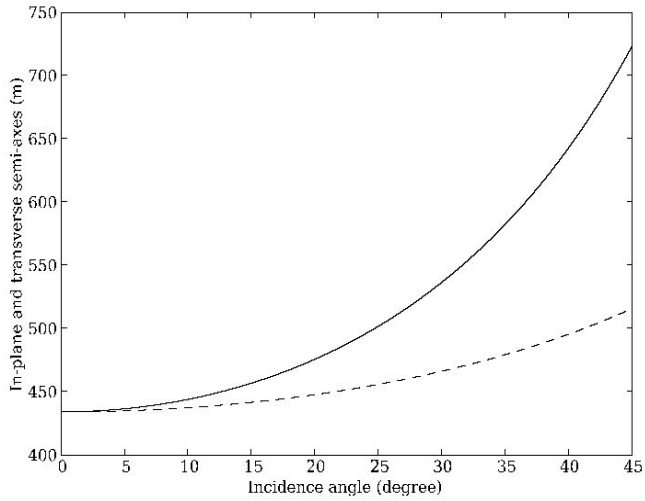

Fig. 2: Variation in the in-plane semi-axis $r^{\parallel}$ (—) and in the transverse semi-axis $r^{\perp}$ (- - -) of the Interface Fresnel zone as a function of the incidence angle θ. The medium configuration is identical to that described in Fig.1.

2 The role of the Interface Fresnel zone in wave propagation

The aim of this section is to investigate the role of the IFZ in the wave reflection process, and more specifically, to point out the consequences of ignoring the IFZ in both forward and inverse modeling of wave propagation.

2.1 Media, model and exact solution

Consider the same configuration as previously. One type of interface between elastic media has been chosen to illustrate the theoretical results. The upper medium has the density $\rho = 2000\,kg/m^3$, the P-wave velocity $V_P = 4000\,m/s$, and the S-wave velocity $V_S = 2000\,m/s$,

whereas the amount in the medium properties through the interface are $\triangle\rho = 400\,kg/m^3$, $\triangle V_P = 1200\,m/s$ and $\triangle V_S = 500\,m/s$. The interface is situated at a distance $z_M = 3000\,m$ from the source-receiver plane. The source spectrum is chosen to be the Fourier transform of a Ricker wavelet with the dominant frequency $f = 32\,Hz$ and the frequency bandwidth $B = 8\,Hz$.

The amplitude of the P-wave reflected from the part of interface which physically contributes to the wave propagation process, and measured at the receiver R, has to be calculated. As the problem under consideration can be viewed as a problem of diffraction by the IFZ, we chose to apply the Angular Spectrum Approach (ASA) [6] combined with the IFZ concept to get the 3D analytical solution to this problem. Using the ASA permits straightforward derivations of the measured amplitude of the reflected wave at the point R. We refer the reader to our previous publication [4] for a detailed description of the procedure. In addition, we used the 3D code OASES to compute accurately synthetic seismograms in media. OASES is a general purpose computer code for modeling seismo-acoustic propagation in horizontally stratified media using wavenumber integration in combination with the Direct Global Matrix solution technique [8]. This 3D code, widely used in the underwater acoustics community, has been thoroughly validated.

2.2 Contribution of the IFZ to the seismic amplitude

Our objective being to evaluate the importance of accounting for the IFZ for calculating the amplitude of the P-wave reflected from the interface, it is instructive to compare the variation in the amplitude obtained with our method, as a function of the incidence angle, with the amplitude predicted by the code OASES which provides the exact solution, and with the amplitude predicted by the classical PW theory (here, the Zoeppritz equations [1]). Fig. 3 depicts these amplitude-versus-angle (AVA) curves for the medium configuration described in Section 2.1.. A geometrical spreading compensation factor equal to $\frac{z_M}{\cos\theta}$ was applied to the predictions of our 3D approximation, and to the synthetic data provided by the 3D code OASES, in order to be compared in a suitable way with the PW predictions.

Inspection of Fig. 3 shows that for small subcritical angles, AVA curves associated with the exact solution and with the PW theory are quite identical. The discrepancies between them do not exceed 1% up to $\theta = 40°$. As the PW reflection coefficient varies smoothly with the incidence angle, the geometrical spreading compensation is sufficient to reduce the amplitude of the reflected wave generated by the point source to the reflected PW amplitude. The effect of the IFZ on the wave amplitude is negligible for small incidence angles in the subcritical region. Between $\theta = 40°$ and the critical angle $\theta_C = 50.28°$, the PW reflection coefficient rapidly increases with the incidence angle, and the geometrical spreading compensation is not sufficient anymore. The discrepancies between the exact curve and the PW reflection coefficient increase with the incidence angle and exceed 70% for θ_C. Therefore, the additional application of the IFZ concept becomes necessary to get the reflected P-wave amplitude. We should point out that as expected, the discrepancies between the exact solution and the PW reflection coefficient strongly increase with decreasing dominant frequency of the source signal [4].

Below and close to the critical angle, the predictions of our approximation better fit the exact solution than the PW reflection coefficient, more particularly between $\theta = 47°$ and θ_C. The discrepancies between the ASA curves and the exact curves do not exceed 5% up to $\theta = 52°$ and are smaller than 1% for θ_C. Nevertheless, with increasing incidence angle, the approximate solution shows increasing discrepancies, in comparison with the exact solution, reaching the maximum value of 22% for $\theta = 56°$. The explanation comes from the fact that we calculated only the reflected wave amplitude, whereas the code OASES provides the amplitude of

the interference between the reflected and the head wavefields. Unfortunately, the contribution of each wavefield to the global amplitude recorded at the receiver cannot be discriminated in the synthetic seismograms. For great postcritical angles, for which the signal relative to the head wave and the signal relative to the reflected wave can be separated in time, our approximation tends to the exact solutions. Our present work is focused on this topic and results will be reported later. Note that general conclusions drawn above which are concerned with the medium configuration described in Section 2.1. are in fact common to other models with lower or stronger impedance contrasts at the interface. We should also point out that, contrary to the PW solution, the predictions of our approximation fit reasonably well the exact solution in the vicinity of the critical angle, whatever the dominant frequency of the source signal [4].

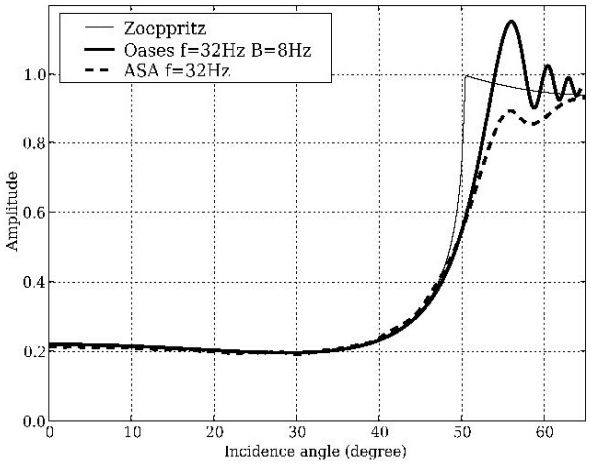

Fig. 3: Variation of the amplitude of the P-wave reflected from a plane interface, as a function of the incidence angle. Comparison between the plane-wave reflection coefficient and the spreading-free amplitudes associated with the exact solution and with the approximate solution. The exact solution is provided by the 3D code OASES, whereas the approximate solution is obtained by applying the Angular Spectrum Approach together with the Interface Fresnel Zone concept. (See text for the description of the medium configuration)

2.3 Estimation bias

Our objective is to evaluate quantitatively the error made on the estimation of the media properties when the IFZ is not taken into account in the inversion procedure. Consider that the real data measured by the receivers are represented in fact by the exact curve provided by the code OASES. The measured reflectivity implicitly accounts for the low-frequency content and the limited bandwidth of data. Nevertheless, one usually assumes that the real curves can be well described by the monochromatic PW theory, and inversion algorithms are therefore based upon the Zoeppritz equations. Which effect on parameter estimation has this assumption?

To answer to this question we developed a quite simple inversion procedure applied to the AVA curves. The properties ρ, V_P, and V_S, of the upper medium being considered as a priori informations, the estimation problem was reduced to the determination of the parameters $\Delta\rho$, ΔV_P, and ΔV_S (i.e. the amount in properties through the interface) which minimize in the least-square sense the cost function D expressing the distance between the real amplitude provided by OASES and the simulated ones represented by the Zoeppritz curve:

$$D = \frac{1}{N} \sum_{n=1}^{N} \left[(Ampl_{OASES})_n - (Ampl_{Zoeppritz})_n \right]^2 , \tag{4}$$

where the parameter N denotes the number of points of the sampled AVA curves. This number was chosen arbitrarily equal to 24. The initial estimates of the parameters $\Delta\rho$, ΔV_P, and ΔV_S, were chosen arbitrarily equal to $100\,kg/m^3$, $100\,m/s$, and $100\,m/s$, respectively. A number of algorithms exist for solving least-square minimization problems. As several parameters had to be recovered simultaneously, we chose the Simplex method. The inversion algorithm was first tested against the PW AVA curve which represented here the measured data. Whatever the initial estimates, the parameters $\Delta\rho$, ΔV_P, and ΔV_S, were identified with a high accuracy, the estimation errors calculated with respect to the real values given in Section 2.1. being below $10^{-4}\%$. We could then trust in the inversion procedure. The inversion algorithm was then tested against the AVA curves provided by the code OASES. This led to the estimations of the amount in the medium properties through the interface reported in Table 1, together with the estimation errors calculated with respect to the real values given in Section 2.1., for various ranges of incidence angles and for the dominant frequency $f = 32\,Hz$ and the source signal bandwidth $B = 8\,Hz$. Firstly, we can note that the media properties are identified with a reasonably good accuracy for typical range of incidence angles in classical AVA studies, e.g. for angles θ varying within the range $[0, 40°]$. The estimation errors are below 1%, except for the parameter $\Delta\rho$ for which the estimation error slightly exceeds 1%. These results which were expected come from the fact that for small incidence angles, the AVA curve provided by the PW theory fits well the exact one (Fig. 3). On the contrary, in the context of wide-angle AVA studies, e.g. for incidence angles θ varying within the range $[0, 55°]$ and containing here the critical angle, the AVA curve provided by the PW theory does not fit the exact one. As a consequence, except for $\Delta\rho$ for which the estimation error is about 4%, the parameters cannot be recovered with accuracy, the estimation errors lying above 14%. Note more particularly the very bad estimation of the parameter ΔV_S, for which the estimation error exceeds 70%. We guess that these estimation errors would increase with decreasing frequency, as the PW theory which has the underlying assumption of infinite-frequency wave propagation cannot fit the exact solution. These preliminary results show that accounting for the frequency content and the limited bandwidth of the source signal seems to be essential to constrain the estimation of media properties in the inversion process for wide-angle AVA studies.

We then tested the previous inversion procedure by considering that the simulated data were now provided by our approximation, i.e. by simply replacing the term $Ampl_{Zoeppritz}$ by the term $Ampl_{approx}$ in the expression 4 of the cost function D. This led to the estimations of the parameters $\Delta\rho$, ΔV_P, and ΔV_S reported in Table 1, together with the estimation errors calculated with respect to the real values given in Section 2.1., for incidence angles θ varying within the range $[0, 55°]$ and for the dominant frequency $f = 32\,Hz$ and the source signal bandwidth $B = 8\,Hz$. Except for $\Delta\rho$ for which the estimation error exceeds 25%, the parameters ΔV_P and ΔV_S are much better estimated than previously with the Zoeppritz formulation, the estimation errors lying below 9%. Note more particularly the good estimation of the parameter ΔV_S, for which the estimation error is below 4%. Considering these preliminary results provided by both forward and inverse modeling of wave reflection, and despite the lack of its high accuracy in the critical angle region, our approximation which accounts for the IFZ seems to be an attractive alternative to the above-mentioned classical methods to extract information relative to bottom parameters from AVO/AVA signals. Our paper emphasizes this feature rather than addressing the question of how to achieve it technically. Nevertheless, the inverse problem needs further investigations and future work will focus on this aspect.

Amount in medium properties	Zoeppritz				Approximation	
	$\theta \in [0, 40°]$		$\theta \in [0, 55°]$		$\theta \in [0, 55°]$	
	estimate	error	estimate	error	estimate	error
$\Delta\rho$ (kg/m^3)	405.6	1.4 %	384.3	3.9 %	298.4	25.4 %
ΔV_P (m/s)	1196.4	0.3 %	1027.2	14.4 %	1302.2	8.5 %
ΔV_S (m/s)	497.3	0.5 %	134.4	73.1 %	519.4	3.9 %

Tab. 1: Estimation of media properties and estimation errors from exact AVA curves for the dominant frequency $f = 32\,Hz$ and the source signal bandwidth $B = 8\,Hz$. The critical angle is $\theta_C = 50.28°$. (See text for the description of the medium configuration)

Conclusion

The goal of the paper was to point out the consequences of ignoring the band-limited property and the low-frequency content of the seismic data in both forward and inverse modeling of wave reflection, as it is traditionnally done for typical AVA (Amplitude-versus-Angle) contexts in geophysical exploration. For this purpose, we have investigated more precisely the role of the Interface Fresnel zone, which physically contributes to the reflected wavefield, in the computation of the reflected P-wave amplitude measured at the receiver, and also in the estimation of media properties from AVA signals. First, AVA curve obtained with the classical plane-wave (PW) theory has been compared with the exact solution provided by the 3D code OASES, and with our approximation which accounts for the Interface Fresnel zone. Our approximation has provided much better data predictions than the PW theory traditionnally used in AVA studies. It has been then shown that assuming that the AVA curves corresponding to real data may be well described by the PW theory leads to inaccurate estimations of the media properties. It was finally suggested that our approximation might be therefore an attractive alternative to classical methods to extract information relative to bottom parameters from AVA signals.

Acknowledgments

The work reported here is the result of discussions and fruitful collaboration with our late friend Dr Eric de Bazelaire during many years. We consider ourselves extremely fortunate to have had the opportunity to work at close quarters with a scientist of his calibre. We also sincerely acknowledge the chairman of the session "Inverse problems in Acoustics", Dr E. Skarsoulis, as well as the audience, to have contributed to honour the memory of Eric who died four days before the beginning of the ICTCA07, by observing a minute's silence after the talk.

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