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Coupled mode analysis of thin micro-perforated panel absorbers

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Running title: Vibroacoustics of thin micro-perforated absorbers.

Abstract: The prediction of the isolating properties of lightweight Micro Perforated Panels (MPP) is a subject that has been intensively studied due to their important applications in a wide range of areas such as building acoustics and the aeronautic, astronautic and automotive industries. MPPs have been mostly considered as rigid structures, accounting only for inertia and neglecting any vibrating effects. However, simulation and experimental studies on thin MPPs have found that the absorbing performance can experience variations in the low frequency range from the results expected assuming a rigid structure. The work presented here is a theoretical and experimental study on the influence of panel vibrations on the sound absorption properties of thin MPP absorbers. Measurements show that the absorption performance generates extra absorption peaks or dips that cannot be understood assuming a rigid MPP. A theoretical model is established that exactly accounts for structural-acoustic interaction between the micro-perforated panel and the backing cavity without restriction on the absorber cross-sectional shape or on the panel boundary conditions. This model is verified experimentally against impedance tube measurements and laser vibrometric scans of the cavity-backed panel response. The effect of micro-perforations on panel-cavity or hole-cavity resonances is revealed through coupled mode analysis.

Keywords: Micro-perforated panels, absorbers, structural acoustics

1. Introduction

Micro-Perforated Panels (MPP) constitute a new type of fibreless absorbers tagged as "next-generation" absorbing materials due to their great potential in comparison with conventional porous materials in a wide range of areas, such as building acoustics and the aeronautic, astronautic and automotive industries. Typical MPP Absorbers (MPPA) are composed of a panel with sub-millimetric holes backed by a rigid cavity, as shown in Figure 1(a). The physical parameters defining the acoustic properties of the MPP are

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the thickness, the size of the perforation and the perforation ratio. It has been shown[1] that the parameter that determines the maximum of absorption and the absorption bandwidth is the panel perforation rate to thickness ratio. Once the configuration parameters have been selected, the backing cavity depth has to be chosen to build the Helmholtz-type resonance.

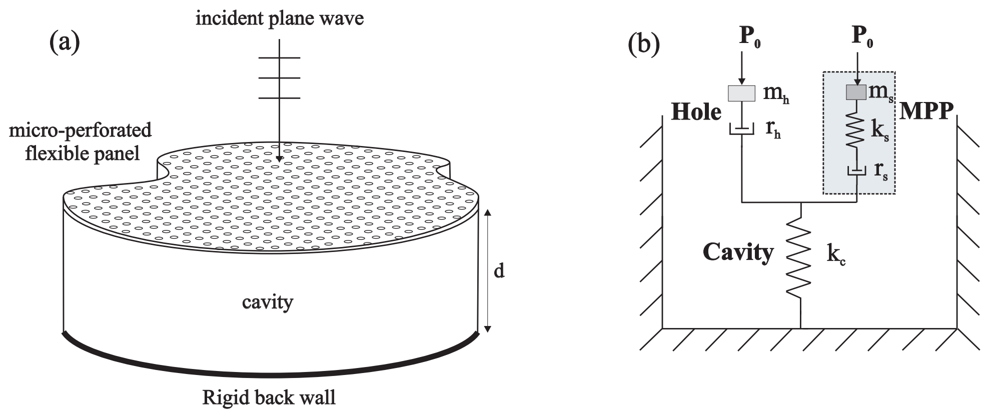

Figure 1 – (a) A cavity-backed MPP absorber with a flexible boundary surface; (b) Mechanical equivalent of an elastic MPP-Cavity absorber.

MPPs have been often considered as rigid structures of infinite extent, accounting only for inertia and neglecting any vibrating effects. However, simulation and experimental studies[2-3] on thin structures have found that the absorbing performance can experience variations in the low frequency range from the results obtained using solely the sound absorbing model proposed by Maa.[1] The objective of this study is to investigate the physical mechanisms that relate the acoustic and elastic properties of thin MPPAs of finite size. Of special interest is the effect of the panel vibrations on hole-cavity resonances and the effect of the microperforations on panel-cavity resonances using coupled mode analysis of the equivalent mechanical system shown in Figure 1(b).

2. Vibroacoustic properties of a thin micro-perforated absorber

Theoretical and experimental studies have been carried out to investigate the absorption and vibroacoustic properties of cavity-backed thin MPPs of arbitrary cross-sectional surface and conservative boundary conditions. As shown in Figure 1(a), the MPPA is excited by a normal incidence plane wave p_0 with angular frequency ω. An analytical model has been established based on modal decomposition of the cavity pressure p (resp. velocity, v) as series of the cross sectional modes of the rigid walled cavity (resp. panel structural modes). Exact continuity conditions are satisfied between the normal particle velocity on the MPP surface, $\tilde{v}$, and the normal air particle velocity in the cavity close to the panel interior side. Assuming that the MPP holes are separated by a distance much lower than the acoustic wavelength, the particle velocity, spatially averaged over each aperture cell, can be expressed as[2] $\tilde{v} = \gamma v + (p_0 - p)/Z$, with $Z = Z_h/\sigma$ the overall acoustic impedance of the rigid MPP, as derived by Maa[1] for circular holes, σ the perforation ratio and $\gamma = 1 - \mathrm{Im}(Z_h)/Z$. For MPPs with sub-millimetric holes, the effects due to viscous losses prevail over inertial effects so that $\gamma \approx 1$. The exact continuity condition and the equation of motion for the flexible panel provide a coupled set of algebraic equations that are solved for the acoustic and structural modal amplitudes to obtain p and v. The MPPA absorption coefficient, $\alpha = 1 - |R|^2$, can thus be calculated in terms of the normal incidence reflection coefficient $R = (Z_1 - Z_0)/(Z_1 + Z_0)$: $Z_0 = \rho_0 c_0$ is the air impedance and $Z_1 = p_0/\overline{\tilde{v}}$ is the input acoustic impedance of the MPPA with $\bar{\bar{v}}$ the air particle velocity averaged over the MPP surface. Expressing p in terms of v, one obtains $Z_1 = (Z + Z_{a,0})/(1 + \gamma \bar{v} Z / p_0)$, with $Z_{a,0} = -jZ_0 \cot(k_0 d)$ the acoustic impedance of the cavity and k_0 the acoustic wavenumber. Assuming single cross-coupling between the first panel mode and the uniform cross-sectional cavity mode, then Z_1 can be expressed as

$$Z_1 = Z_{a,0} + \frac{Z Z_p}{Z + Z_p} \tag{1}$$

with Z_p the impedance of the first panel mode. For a rigid MPPA ($Z_p \to \infty$), Z_1 reduces to $Z + Z_{a,0}$ whose imaginary part is zero-valued at the Hole-Cavity (HC) resonance of the rigid MPPA, e.g., the Helmholtz resonance, given by $\omega_{HC}^2 = k_c / m_h$, with $k_c = \rho_0 c_0^2 / d$ the acoustic equivalent stiffness of the cavity and m_h the "effective mass" per unit area of the MPP.[1] For a non-perforated panel ($Z \to \infty$), then Z_1 reduces to $Z + Z_p$ whose imaginary part is zero-valued at the first Panel-Cavity (PC) resonance given by $\omega_{PC}^2 = \omega_1^2 + \Delta_{PC}^2$ with ω_1^2 the natural frequency of the first panel mode and $\Delta_{PC}^2 = k_c / m_p$ the panel-cavity coupling coefficient.

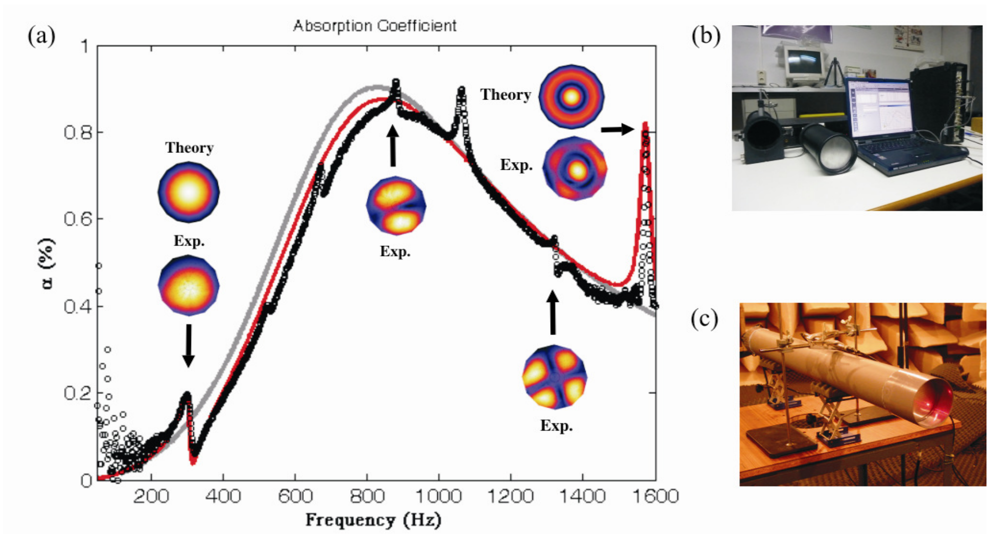

Figure 2 – (a) Sound absorption coefficient of a thin MPPA: predicted assuming a rigid (grey) or an elastic (red) MPP and measured (circles) and comparison between the measured and predicted velocity distribution of the disk panel in relation with local maxima of the sound absorption coefficient; Experimental set-ups used for measuring the MPPA absorption coefficient (b) and the MPP velocity response when backed by a cavity (c).

Experiments have been conducted on a MPPA made up of a thin micro-perforated aluminium disk of radius 50 mm and thickness 0.5 mm, backed by a rigid cylindrical cavity of depth 45 mm. The disk is uniformly perforated by circular holes with diameter 0.5 mm and perforation ratio 0.78%. The absorption coefficient is measured in an impedance tube shown in Figure 2(b) using the two-microphones method. A good agreement is observed in Figure 2(a) between the experimental results and the theoretical predictions provided that the elastic behaviour of the panel is accounted for in the model. Extra absorption peaks at 298 Hz and 1572 Hz are clearly observed and well predicted by the vibroacoustic model. They correspond to PC-controlled resonances that increase the acoustic resistance of the MPPA. Vibroacoustic measurements of the MPP vibrations have been performed using a laser vibrometer with the scanning head focused on the MPP through the thick transparent rear face, as shown in Figure 2(b). The absorption peaks observed at 298 Hz and 1572 Hz are clearly the (0,0) and (1,0) volume-displacing PC resonances of the simply-supported disk panel. Extra peaks at 881 Hz and 1319 Hz are due to non axisymmetric disk modes excited by non-uniform shear stresses that occur along the disk circumference.

3. Coupled mode analysis

From Eq. (1), the MPPA can be viewed as a parallel connection between a mass-spring damped oscillator and a mass-resistance element coupled in series through the air cavity stiffness, k_c, and driven by the impinging sound wave of amplitude p_0, as sketched in Figure 1(b). The mass-spring damped element, m_p, k_p, r_p, describes the panel average velocity $\bar{v}$ when vibrating on its first mode. The mass-resistance element, m_h, r_h, is due to the effective air mass moving in-phase through the holes with velocity v_h. Put in coupled mode form, the equations of motion read

$$m_p\dot{\bar{v}} + r_p\bar{v} + k_p\bar{u} = p_0 - k_c\left(u_h + \bar{u}\right) \tag{2}$$

$$m_h\dot{v}_h + r_h v_h = p_0 - k_c\left(u_h + \bar{u}\right) \tag{3}$$

with $k_p = \omega_1^2 m_p$, $r_p = \eta_1\omega_1 m_p$ and $r_h = \mathrm{Re}(Z)$. η_1 is the panel modal damping and m_p is the modal mass of the first panel mode. The upper-dot denotes a time derivative. $\bar{u}$ and u_h are respectively the panel average displacement and the effective air mass displacement. The resonance modes and frequencies for the undamped coupled system are found setting p_0, η_1 and r_h to zero in Eqs. (2-3) and seeking solutions of the form $\bar{u} = u_{0p}\,\mathrm{e}^{\mathrm{j}\Omega t}$ and $u_h = u_{0h}\,\mathrm{e}^{\mathrm{j}\Omega t}$, thus yielding a set of two homogeneous equations for the mode amplitudes u_{0p} and u_{0h}:

$$\left(\omega_{PC}^2 - \Omega^2\right)u_{0p} + \Delta_{PC}^2\,u_{0h} = 0 \tag{4}$$

$$\omega_{HC}^2\,u_{0p} + \left(\omega_{HC}^2 - \Omega^2\right)u_{0h} = 0 \tag{5}$$

The system (4-5) has nontrivial solutions if its determinant vanishes, thus providing the following expressions for the two resonance frequencies, $\Omega_\pm$, and the components of each resonance mode

$$\Omega_\pm^2 = \frac{\omega_{PC}^2 + \omega_{HC}^2}{2} \pm \sqrt{\left(\frac{\omega_{PC}^2 - \omega_{HC}^2}{2}\right)^2 + \Delta_{PC}^2\,\omega_{HC}^2} \tag{6}$$

$$u_{0p}^\pm = \frac{\Omega_\pm^2 - \omega_{HC}^2}{\omega_{HC}^2}\,u_{0h}^\pm \tag{7}$$

A transfer factor, $F_{\Omega_\pm}$, is defined between the two resonance modes such that $F_{\Omega_\pm}^{-1} = 1 + \omega_{PC}(\omega_{PC} - \omega_{HC})^2 / (2\omega_{HC}\Delta_{PC}^2)$: it determines the fraction of energy transferred between the modes. $F_{\Omega_\pm} = 1$ occurs for a critical value of the perforation ratio given by $\sigma_c \approx 2.2\rho_0 t_h \omega_{PC}^2 / (m_p \Delta_{PC}^2)$ and for which all the energy is interchanged between the resonant states ($\omega_{PC} = \omega_{HC}$). From Eq. (6), the resonances frequencies then read $\Omega_\pm^2 = \omega_{HC}^2 \pm \Delta_{PC}\,\omega_{HC}$. They correspond to the frequencies of maximum absorption on the red dashed curves in Figure 3(a), located symmetrically apart from the Helmholtz resonance (grey dashed curve). If $F_{\Omega_\pm} << 1$, the resonance modes are little affected by one another. Weak coupling occurs when $\sigma << \sigma_c$ or when $\sigma >> \sigma_c$. The second case is often encountered in practice, especially for the MPPA used in the experiment. From Eq. (6), the upper and lower resonances are approximated by

$$\Omega_\pm^2 \approx \omega_{HC,PC}^2 \pm \frac{m_h}{m_p} \frac{\omega_{HC}^4}{\omega_{HC}^2 - \omega_{PC}^2} \tag{8}$$

When σ increases, the upper resonance frequency Ω_+ stays above (and tends towards) the HC resonance, as shown from the dash-dotted curves on Figure 3(a), and the lower resonance frequency Ω_- tends towards the first uncoupled panel mode frequency ω_1, as seen from the red dash-dotted and red solid curves on Figure 3(a) and from Eq. (8). Close to the HC resonance, Eq. (7) shows that the panel and air particles move in phase but with a large air-frame relative velocity due to $|u_{0p}^-| << |u_{0h}^-|$ whereas, close to the panel resonance, the panel and the air particles move in opposite phase with an amplitude ratio given by $|u_{0p}^- / u_{0h}^-| = 1 - \omega_1^2 / \omega_{HC}^2$. Figure 3(b) shows that the upper and lower resonance frequencies calculated from Eq. (6) well agree with those obtained from the vibroacoustic model. Near $\sigma = \sigma_c$, the upper and lower resonances of the MPPA are strongly coupled as each part of the PC- and HC-controlled modes has the same amount of energy. When $\sigma << \sigma_c$, the upper and lower resonances of the MPPA are respectively PC- and HC-controlled whereas, when $\sigma >> \sigma_c$, they are HC- and panel-controlled, weakly coupled in both cases.

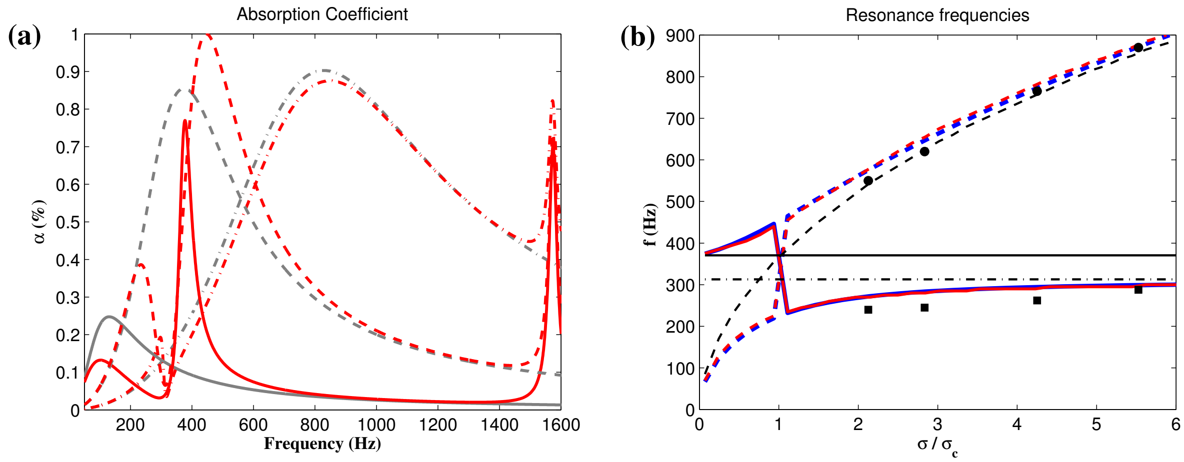

Figure 3 – (a) Simulation results on the influence of the perforation ratio on the sound absorption coefficient of a rigid (grey) or flexible (red) MPP absorber: $\sigma(=0.02\%) < \sigma_c$ (solid); $\sigma(=0.16\%) = \sigma_c$ (dashed) ; $\sigma(=0.78\%) > \sigma_c$ (dash-dotted); (b) Influence of the normalised perforation ratio σ / σ_c on the first PC-controlled (solid) and HC-controlled (dashed) resonant frequencies of a thin MPPA: coupled mode analysis (blue) and vibro-acoustic model (red). Black reference curves: HC resonance (dashed); first PC resonance (solid) and first panel resonance (dash-dotted). Upper (circles) and lower (squares) measured resonance frequencies.

4. Conclusions

This study shows that the air-frame relative velocity is a key factor that alters the input acoustic impedance of thin MPPAs and generates extra absorption peaks or dips that are not observed when dealing with rigid MPPAs. Coupled mode analysis reveals that the two first absorption peaks of thin MPPAs are related to either PC-, HC- or panel-controlled resonances of the panel-hole-cavity system, depending on whether the effective air mass of the perforations is greater or lower than the panel first modal mass.

References and links

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