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Micro-perforated panels for silencers in ducted systems

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Summary
Classical porous materials are extensively used in ducted systems such as heating, ventilation and air conditioning, or in automotive exhaust systems. However, there are many difficulties in using such passive materials in these situations. The available space to install the control device is often limited and the surrounding environment can be hostile. In many applications, they are subject to high air flow velocities and temperatures that cause their deterioration and a lack of long-term security conditions. Micro-Perforated Panels (MPPs) are promising solutions in order to achieve noise reduction in such difficult environments. They consist of a thin panel with sub-millimetric perforations situated in front of an air cavity creating a resonance-type absorber. The attenuation of sound is due to friction losses through the holes at the Helmholtz-type cavity resonance. These absorbers are non-combustible, ecologically friendly, cleanable, and their efficiency bandwidth is tuneable. By a proper selection of the constitutive physical parameters, it is possible to obtain a relatively broadband attenuation partition without the introduction of fibrous materials. These devices were initially proposed by D. Y. Maa and were mainly conceived for applications in room acoustics. In this work, we present a theoretical model for the prediction of the acoustic properties of a cylindrical silencer based on the Multi-Modal Propagation Method, that calculates the propagation of sound in ducts with varying cross-section and with either rigid, locally- or non-locally reacting boundary conditions. This method is adapted to analyse the absorbing properties of a circular duct lined with a MPP treatment.

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1. Introduction

The use of classical porous materials in duct dissipative silencers is widespread mostly because of their broadband absorption characteristics. Nevertheless, these devices may have to perform in hostile environments and are often subject to grazing flows, high temperatures or dust particles, that are responsible of their rapid physical and acoustical deterioration. Under certain conditions, they imply maintenance and hygienic issues in the working environment such as in food industries or in hospitals. To minimize these effects, porous materials are usually covered by perforated metal panels. The influence of the perforations in the acoustic performances of the whole control device has been extensively studied [1]. The use of panels with micro-perforations is relatively more recent. Initially studied by D. Y. Maa [2-4], MPPs not only protect the porous lining materials, but significantly increase the absorption characteristics by converting the incident acoustic energy into viscous losses through the micro-perforations, the performance of which is determined by the ratio of the aperture radius to the viscous boundary layer thickness [3-4]. The effective dissipation will then depend strongly on the proper selection of the physical parameters constituting the panel. These materials constitute an alternative to achieve noise reduction in difficult environments as they are non-combustible, easy to clean and maintain, and constitute compact devices that can fit into the limited available space. They have been mostly applied as panel absorbers in room acoustics, but they are rapidly extending to automotive and

aeronautic applications such as silencers in ducted systems.
Wu [5] used a simplified analytical model to estimate the insertion loss of MPP duct silencers and concluded that the perforation ratio was the main factor influencing the performance. Allam and Åbom [6] studied the effects of an axial flow on the acoustic impedance of silencers using FEM, and verified the results experimentally using the two-port microphone method. Wang *et al*. [7] have presented a two-dimensional model to study a silencer made up of an expansion chamber with two-side cavities covered by micro-perforated vibrating plates that reflect the sound upstream. Glav and Färm [8] described the sound field inside a cylindrical silencer with MPP baffles as modal expansions using Bessel functions and modelled the acoustic properties of the MPP silencer in terms of a transfer matrix to calculate the TL. All these publications have studied the sensitivity of the silencers acoustic performances to changes in the physical layout conditions.
In this work, we present an analytical model based on the Multi-Modal Propagation Method (MMPM) [9], able to describe propagation in duct equipped with non-uniform liners or with varying cross-sections. The acoustic predictions are verified against measurements in laboratory conditions, and guidelines are provided for the optimal selection of the liner parameters.

2. Theoretical model

The theoretical model presented in this section is an analytical formulation generalizing the MMPM developed by Bi *et al.* [9] to non-local boundary conditions satisfied through a MPP liner enclosing an annular expansion chamber. It provides a physical analysis of the phenomena involved and a low computational cost to study the parameters influence. The system under study is schematized in Fig. 1, that shows an infinite duct with an inner circular cross section of radius R_i and an expansion chamber with outer radius R_o. It contains four different axial sections that have to be treated separately. A source section is situated at $z = 0$, and a lined section with a micro-perforated cylindrical duct that extends from $z = l_1$ to $z = l_2$. Although the theoretical treatment is presented here without considering the presence of a uniform axial flow, this could be added without greatly modifying the general formulation [10]. The pressure and velocity fields for the inner pipe are expressed in terms of the normal modes of a rigid duct, as follows

$$p(r,\theta,z) = \sum_m \sum_n P_{mn}(z)\psi_{mn}(r,\theta) = \mathbf{\Psi}^{\mathrm{T}}\mathbf{P}, \quad (1)$$

$$v(r,\theta,z) = \sum_m \sum_n V_{mn}(z)\psi_{mn}(r,\theta) = \mathbf{\Psi}^{\mathrm{T}}\mathbf{V}, \quad (2)$$

where $P_{mn}(z)$ and $V_{mn}(z)$ are the pressure and velocity modal amplitudes of the normal modes respectively (expressed in matrix notation as the modal vectors $\mathbf{P}$ and $\mathbf{V}$ respectively) and $\psi_{mn}(z)$ are the hard-walled duct normal modes, given by

$$\psi^i_{mn}(r,\theta) = e^{im\theta}\frac{J_m\left(\kappa^i_{mn} r\right)}{\sqrt{\Lambda^i_{mn}}}. \quad (3)$$

In Eq. (3), $J_m(\kappa_{mn} r)$ is the Bessel function of the first kind of order m and the radial wave numbers κ^i_{mn} correspond to the zeros of the normal derivative of the Bessel function at the inner duct wall. $\Lambda^i_{mn} = 2/R_i^2 \int_0^{R_i} J_m^2\left(\kappa^i_{mn} r\right) r dr$ is the normalization factor. For the outer annular section, the normal modes take the expression [11] (except for the plane wave, $\psi^o_{00} = 1/\sqrt{\Lambda^o_{00}}$)

$$\psi^o_{mn}(r,\theta) = \frac{e^{im\theta}}{\sqrt{\Lambda^o_{mn}}}\left[J_m\left(\kappa^o_{mn} r\right) - \frac{J'_m\left(\kappa^o_{mn} R_o\right)}{Y'_m\left(\kappa^o_{mn} R_o\right)} Y_m\left(\kappa^o_{mn} r\right)\right], \quad (4)$$

where Y_m is the Neumann function and the radial wavenumbers for the outer duct cavity, κ^o_{mn}, are numerically calculated using a Newton method to solve the determinant of a two-equation homogeneous system obtained when imposing hard-walled boundary conditions to the annular modes at $r = R_i$ and $r = R_o$.
To obtain a solution for the field quantities, it is necessary to combine the linearised equations of motion (Euler, mass conservation and state equation) with the corresponding rigid or impedance boundary conditions. The solution is expressed as a sum of modal amplitudes for each rigid or lined section. Applying the appropriate conditions (pressure and velocity continuity at the junctions, pressure continuity and velocity jump across the source section, jump of pressure across the MPP), one obtains a global solution for the modal amplitudes at any position inside the duct.
The modal pressure vectors of the outer and inner duct sections then satisfy the following second-order differential equations

$$\mathbf{P}^{o''}(z) + \left[k^2\mathbf{I} - \kappa^{o^2}_{mn}\mathbf{I} - \frac{ik}{\overline{Z}_{\mathrm{MPP}}}\mathbf{C}^o\right]\mathbf{P}^o(z) + \frac{ik}{\overline{Z}_{\mathrm{MPP}}}\mathbf{C}^{io^{\mathrm{T}}}\mathbf{P}^i(z) = \mathbf{0}, \quad (5)$$

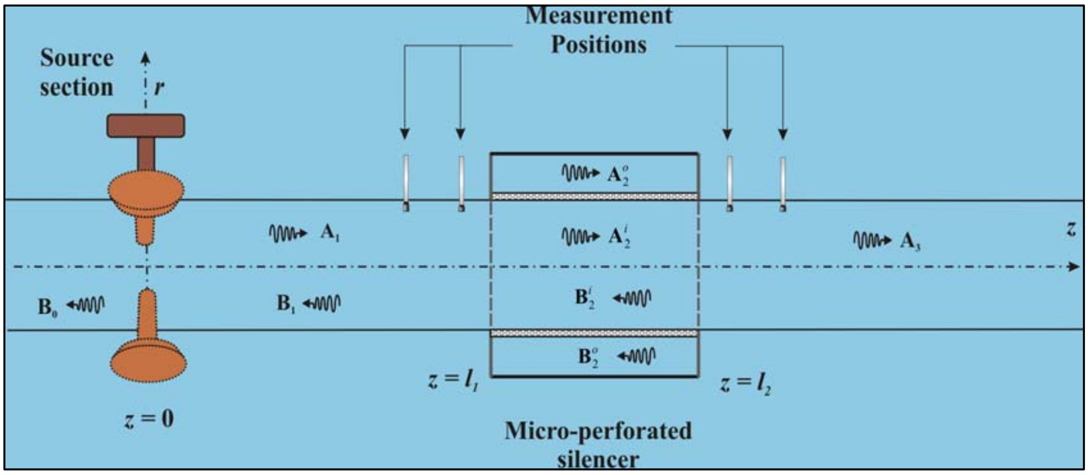

Figure 1. Diagram of the physical set-up

$$\mathbf{P}^{i\,\prime\prime}(z)+\left[k^2\mathbf{I}-\kappa_{mn}^{i^2}\mathbf{I}-\frac{ik}{\overline{Z}_{\text{MPP}}}\mathbf{C}^i\right]\mathbf{P}^i(z)+\frac{ik}{\overline{Z}_{\text{MPP}}}\mathbf{C}^{io}\mathbf{P}^o(z)=\mathbf{0}, \tag{6}$$

where $\mathbf{I}$ is the identity matrix and $\mathbf{C}^i$, $\mathbf{C}^o$ and $\mathbf{C}^{io}$ are matrices describing the modal cross-coupling between the inner, outer and inner-outer duct modes through the MPP surface [12]. We provide the following expression for the elements of $\mathbf{C}^i$ which would be the only coupling matrix in the case of a locally-reacting MPP:

$$\mathbf{C}^i_{m,m,m',n'}=\frac{2\pi\, J_m\left(\kappa^i_{mn}R_i\right)J_{m'}\left(\kappa^i_{m'n'}R_i\right)}{\sqrt{\Lambda^i_{mn}\Lambda^i_{m'n'}}}\delta_{mm'}. \tag{7}$$

The interested reader may refer to [12] for $\mathbf{C}^o$ and $\mathbf{C}^{io}$ in the general case. In (5-6), $\overline{Z}_{\text{MPP}}= Z_h/(\sigma\rho_0 c)$ is the overall MPP specific impedance, obtained from the work of Maa [3, 4] in terms of Z_h, the impedance of the MPP holes,

$$Z_h=\frac{32\eta t_h}{d_h^2}\left[\sqrt{1+\frac{k_h^2}{32}}+\frac{\sqrt{2}}{32}k_h\frac{d_h}{t_h}\right]+ j\rho_0\omega\, t_h\left[1+\left(9+\frac{k_h^2}{2}\right)^{-1/2}+\frac{8}{3\pi}\frac{d_h}{t_h}\right], \tag{8}$$

with σ the perforation ratio, d_h the diameter of the circular hole, t_h the panel thickness, η the dynamic viscosity of the air and $k_h=(d_h/2)/r_{\text{visc.}}(\omega)$, the perforate constant, e.g. the ratio of the hole radius to the viscous boundary layer thickness, $r_{\text{visc.}}(\omega)=\sqrt{\eta/\rho_0\omega}$.

Eqs. (5-6) can be arranged as a single second-order differential equation of the modal coefficients of the propagation vector $\mathbf{P}(z)=\left\{\mathbf{P}^o;\mathbf{P}^i\right\}$, as follows

$$\mathbf{P}''+\mathbf{MP}=\mathbf{0},\quad l_1<z<l_2. \tag{9}$$

where the matrix $\mathbf{M}$ takes the expression

$$\mathbf{M}=\begin{pmatrix} k^2\mathbf{I}-\kappa_{mn}^{o^2}\mathbf{I}-\dfrac{ik}{\overline{Z}_{\text{MPP}}}\mathbf{C}^o & \dfrac{ik}{\overline{Z}_{\text{MPP}}}\mathbf{C}^{io\,\text{T}} \\ \dfrac{ik}{\overline{Z}_{\text{MPP}}}\mathbf{C}^{io} & k^2\mathbf{I}-\kappa_{mn}^{i^2}\mathbf{I}-\dfrac{ik}{\overline{Z}_{\text{MPP}}}\mathbf{C}^i\end{pmatrix}. \tag{10}$$

If an eigenvalue decomposition is performed on the $\mathbf{M}$ matrix,

$$\mathbf{M}=\mathbf{X}\boldsymbol{\Lambda}\mathbf{X}^{\text{H}}, \tag{11}$$

the inner and outer modal pressure amplitudes can be expressed as a function of the eigenvectors and eigenvalues. The vector of the modal amplitudes for all the sections outlined in Fig. 1 is then composed of six unknown coefficients, $\mathbf{U}^{\text{T}}=\left\{\mathbf{B}_0,\mathbf{A}_1,\mathbf{B}_1,\mathbf{A}_2^o,\mathbf{B}_2^o,\mathbf{A}_2^i,\mathbf{B}_2^i,\mathbf{A}_3\right\}$, that can be determined formulating the proper conditions for continuity of the pressure and axial velocity modal components at the junctions between the different sections, on the annular interface and across the source distribution. We obtain a system of algebraic equations that can be solved to determine the modal amplitudes and then the pressure and velocity fields at any positions inside the duct.

3. Experimental procedure

The theoretical model has been verified against a set of measurements carried out in laboratory conditions. The physical set-up has been designed and installed in a semi-anechoic environment to characterize the acoustic properties of an annular micro-perforated silencer. The test bench consists of a 1 cm-thick PVC tube of length 2.70 m with a 9.5 cm inner diameter, made up of different sections, as it can be seen in Fig. 1. We find a source module with one loudspeaker connected to one duct end and responsible for the generation of a random signal from 50 Hz to 3 kHz. Then, there is a first measurement hard-walled section in which two microphones are used to perform wall-pressure measurements. It is followed by the test section, in which the cylindrical silencer is installed. It is connected to a second measurement hard-walled section identical to the first one. To end with, we find the duct outlet, that can be fitted to simulate different boundary conditions, such as a rigid or an anechoic termination. For the measurements carried out in this work, we have connected a second loudspeaker that allows us to obtain a second estimation of the (right-to-left) acoustical properties of interest. A photograph of this configuration is shown in Fig. 2.

Figure 2. Photograph of the standing wave duct installed in the semi-anechoic chamber.

The tested device was constituted of two co-axial cylinders. The inner one is micro-perforated and is made of aluminium with a Young modulus of $6.9\times10^{10}\,\mathrm{N/m^2}$, a mass density of $2700\,\mathrm{kg/m^3}$ and a Poisson ratio of 0.33. The cylinder thickness is 0.5 mm, and the holes axis are separated by a distance of 5 mm, so that the perforation ratio is 0.78 %. The holes diameter is 0.5 mm. Details of the inner MPP cylinder can be seen in Fig. 3. The holders that maintain the silencer to the rigid duct sections can also be appreciated. They were 3D-printed from ABS rigid polymer material.

Figure 3. Photograph of the inner micro-perforated cylinder.

The MPP silencer has an internal radius of 4.75 cm, with a cavity depth of 2.8 cm between the internal and external ducts. The total length of the test partition was 0.4 m.

The measurement procedure has been carried out following the four-microphones approach [13]. Starting from one active loudspeaker on one side, a random signal is generated that propagates along the duct to the rigid measurement sections where a couple of calibrated sensors are situated. A close view of the microphone positions on the standing wave tube is presented in Fig. 4. The transfer functions between the input signal and the four measurements positions are acquired using the OROS (type OR38) multi-channel acquisition system. If we assume that the first microphone is situated at $x=0$, δ_1 and δ_2 are the separation distances between the first and second microphone, and the third and fourth microphones respectively, D_P the length of the partition, and l_1 and l_2 are the distances between the sample and the first and third microphones, then the sound pressure acquired at the measurement positions can be expressed as

$$p_1 = p(x_1=0) = Ae^{-jkx_1} + Be^{jkx_1}, \qquad (12)$$

$$p_2 = p(x_2=\delta_1) = Ae^{-jkx_2} + Be^{jkx_2}, \qquad (13)$$

$$p_3 = p(x_3 = l_1 + D_\mathrm{P} + l_2) = Ce^{-jkx_3} + De^{jkx_3}, \qquad (14)$$

$$p_4 = p(x_4 = l_1 + D_\mathrm{P} + l_2 + \delta_2) = Ce^{-jkx_4} + De^{jkx_4}. \qquad (15)$$

This is a system of equations that allows the determination of the pressure and velocity in the two rigid measurements sections in terms of the complex amplitudes A, B, C and D.

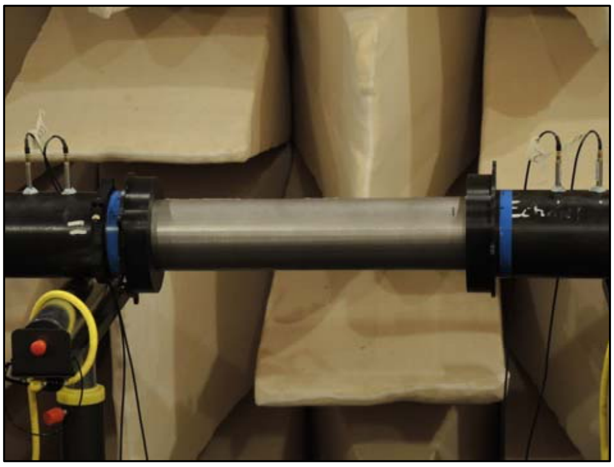

Figure 4. Close view of the two-microphones installed before and after the MPP section (without the annular duct).

We could then proceed with the calculation of the acoustic estimators of interest for the problem. In particular, the reflexion coefficient at the source side of the partition takes the expression

$$R(x=l_1)=\frac{B}{A}e^{2jkl_1}=\frac{H_{12}-e^{-jk\delta_1}}{e^{jk\delta_1}-H_{12}}e^{2jkl_1}, \quad (16)$$

where $H_{12}=p_2/p_1$ is the measured transfer function between the first and second microphones. The transmission coefficient can be calculated as

$$\tau=\frac{C}{A}e^{-jkD_P}=H_{13}\frac{e^{jk\delta_2}-H_{34}}{e^{jk\delta_1}-H_{12}}\frac{\sin(k\delta_1)}{\sin(k\delta_2)}e^{jk(l_1+l_2)}, \quad (17)$$

where $H_{13}=p_3/p_1$ and $H_{34}=p_4/p_3$, measured between the pairs 1-3 and 3-4 of microphones. In the next section, the absorption and transmission properties estimated with this procedure will be compared to the theoretical models outlined above.

4. Comparison with the measured acoustic properties of the silencer

We start the comparison by computing the power dissipated by the micro-perforated liner. This quantity takes into consideration both absorption and transmission properties of the noise control device, and constitutes a representative indicator that could be used for a proper optimization of the physical parameters involved in each of the sub-systems of the partition. It is defined as the difference between the absorption and transmission coefficients, $\alpha-\tau$, and represents the fraction of incident power dissipated by the MPP system. It is also called the absorptivity.

The experimental averaged values obtained considering left-hand and right-hand excitations are presented in Fig. 5 in blue. These results are plotted with the values predicted by two different analytical models. First, we have plotted in red the prediction considering annular propagation conditions in the expansion chamber, as described in Section 2 of this work. We also used the MMPM formulation, but on a simplified configuration that avoids the calculation of the normal modes and frequencies of an annular rigid duct. It consists of a cylindrical duct with a locally reacting lined section uniformly distributed along the duct axis (micro-perforated sheet over honeycomb cells or porous materials filling the annular cavity). The results obtained with this formulation are presented in green in Fig. 5.

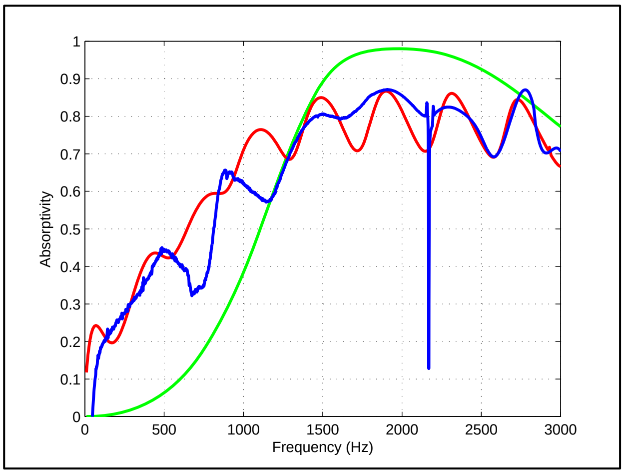

Figure 5. Experimental (blue) values for the dissipated power and comparison with those predicted for a locally reacting MPP liner (green) and when considering annular propagation (red).

As it can be clearly appreciated, the simplified locally reacting liner model provides results that constitute a very rough approximation to the real measured values. It underestimates the absorptivity in the low frequency range below the Helmholtz-type resonance, whereas it clearly overestimates the measured absorptivity values above 1.5 kHz. On the other hand, the model considering propagation within the annular expansion chamber provides quite good approximation over the whole frequency range, with local minima due to the occurrence of axial standing waves in the annular cavity.

We could also compare the transmission properties estimated from the two models by plotting the predicted TL results against those measured. The results presented in Fig. 6 show the experimental TL values in blue, those predicted by the locally reacting liner model in green and those obtained when considering propagation in the annular duct in red. We confirm the same trend that we have already found for the dissipation. The complete model considering annular propagation provides much more accurate results that those obtained with the simplified locally reacting liner formulation. Note that the absorptivity and TL values measured above 2.2 kHz, the first duct cut-on frequency, were obtained from an extension of the 2 x 2 microphones method described in Sec. 2 to a 2 x 6 microphones configuration able to handle the reflexion and transmission contributions of the (±1,0) mode of the inner duct.

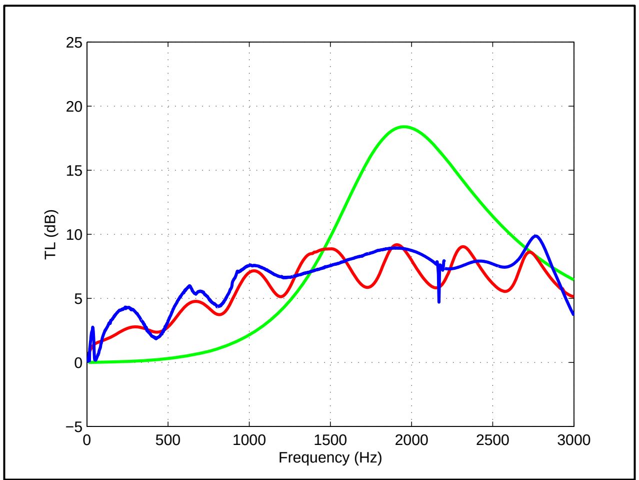

Figure 6. Experimental (blue) values of TL and comparison with those obtained for a locally reacting liner (green) and by considering annular propagation in the annular cavity (red).

5. Conclusions

In this work we have studied the acoustic performance of micro-perforated partitions in ducted systems where the use of porous materials seems problematic. We have formulated an analytical approach as a generalization of the Multi-Modal Propagation Method. This general formulation can be particularized to two different models. A simplified version considers a cylindrical duct with a locally reacting lined section, whereas a more complete description includes sound propagation in the annular duct. We have compared the predicted absorptivity and TL values with the experimental ones acquired in a standing wave tube facility in laboratory conditions. We found that the formulation including annular propagation conditions provides reasonably accurate results, whereas the locally-reacting liner clearly overestimates the measured absorptivity and TL values over the frequency range of maximum performance. Further work will be directed towards the optimization of the MPP physical parameters including cost functions taking into account acoustic indicators adapted to different ducted noise control configurations.

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