NUMERICAL METHODS FOR MULTIPLE SCATTERING OF ULTRASOUNDS IN RANDOM MEDIA

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Abstract

The mathematical basis of theories for studying multiple scattering is well understood. However, the real domain of validity of these methods is still not clearly established. This paper presents the frame for a numerical validation of a classical method, the Independent Scattering Approximation, detailing specific implementation and signal processing used to obtain the parameters of an equivalent homogeneous medium. The setup is illustrated with a problem already well studied in an experimental way, but presenting strong difficulties for the numerical simulation.

Introduction

Let us consider the propagation of ultrasounds in an heterogeneous medium, where the wavelength and the size of heterogeneities are similar. In this case, the wave field is considered as the superposition of a coherent field and of an incoherent field. The coherent field resists to the averaging on disorder, and it can be interpreted as waves propagating in an equivalent homogeneous effective medium. Moreover, the coherent field is progressively dispersed and attenuated, even if the propagation media have no intrinsic dispersion and attenuation.

The aim of the treatment of multiple scatterint is to define an equivalent homogeneous effective medium to the real heterogeneous medium. If the heterogeneities are statistically homogeneous and do not depend on the incident wave vector, the effective wavenumber satisfies $k_{eff}^2(\omega) = k_0^2 - \Sigma(\omega)$, where k_0 denotes the wavenumber of the host medium, ω is the angular frequency, and $\Sigma(\omega)$ contains all information about multiple scattering and must be determined.

The *Independent Scattering Approximation* (ISA) is a classical method to determine the parameters of the equivalent homogeneous medium. It takes into account the single scattering [1]: for each scatterer, the surrounding field is considered as not perturbed by the other scatterers, and there is no correlation between the scatterers. Then, Σ follows from physical and geometrical features of the scatterers, and also from the density n of scatterers in the host medium. To respect the hypotheses of ISA, the scatterers have to be far one from the others (n has to be low), and the impedance contrast between scatterers and the host medium must be weak. When one of these hypotheses is relaxed, the ISA may still provide accurate results. For instance, it has been experimentally shown to describe well cases of steel rods embedded in water [2]: the surface concentration of scatterers did not exceed 15%, but the impedance contrast between steel and water was high.

The goal of our work is to examine the precision of the ISA, especially outside its theoretical domain of validity. To our knowledge, no systematic study has been dedicated to this subject. Our methodology is purely numerical, based on direct numerical simulations and on signal-processing tools. This methodology avoids the limitations of the ISA. It is much faster and less expensive than real experiments, allowing also much finer measures.

Direct numerical simulations

Numerical methods

The aforementionned configuration, studied experimentally in a two-dimensional (2D) setup [2], is chosen to illustrate our approach. The 2D computations are performed on uniform Cartesian grids with spatial mesh size $\Delta x = \Delta y$. A velocity-stress formulation of acoustics (in fluid) and elastodynamics (in solid) is followed. The linear first-order hyperbolic systems so-obtained are integrated by the classical second-order Lax-Wendroff scheme. The main source of numerical dispersion and numerical diffusion follows from the numerical propagation in the fluid. A plane-wave analysis bounds these artefacts by their theoretical values in the homogeneous fluid medium in 1D (see section Results).

To couple the computations done on the fluid matrix and on the solid inclusions, one uses an interface method [5]. This numerical method describes accurately the geometry of interfaces, avoiding the spurious diffractions induced by a naive stair-step description of interfaces. Moreover, the fluid-solid boundary conditions are strongly enforced. Lastly, it maintains the global pre-

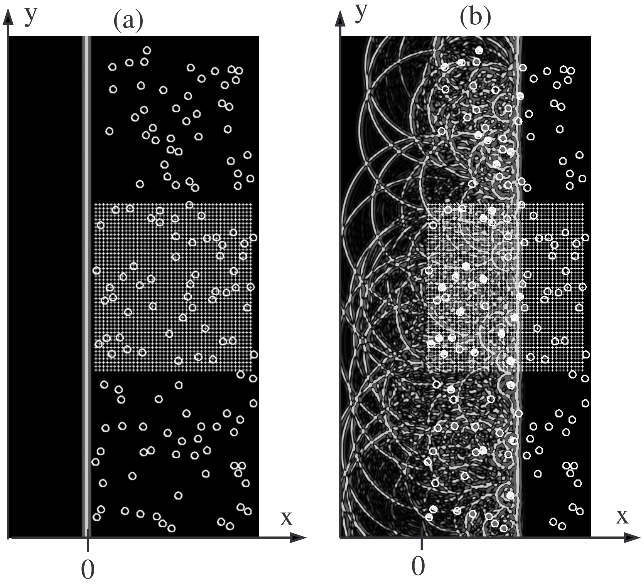

Figure 1: Snapshots of the acoustic pressure at the initial instant (a) and after 10000 steps (b), for a concentration of scatterers of 0.12 rods/mm^2. The regular grid denotes the location of receivers.

cision of the scheme of integration, despite the non-smoothness of solutions across interfaces.

Numerical setup

The heterogeneous medium under study is made of circular rods of steel randomly embedded in water. The intrinsic attenuation of the media are not taken into account. The size of the computational domain is respectively 3 cm along x and 6 cm along y. The mesh size is $\Delta x = 7.5\,\mu$m. The 0.4 mm radius rods are randomly distributed on a 2 cm-wide subdomain (i.e around 3 mean free path). An exclusion length of $4\,\Delta x$ between each inclusion is ensured. This correlation between scatterers, contradicting one hypothesis of the ISA, is not too penalizing [2]. For each simulation, the right-going incident plane wave is a Ricker centred at 2 MHz (wavelength in water: $\lambda_0 = 0.75$ mm). The CFL number is 0.6 in the steel; for stability reasons, the CFL is not optimal in water.

A regular array of receivers with 45 lines and 42 columns is put on the domain every $\Delta x_R = \Delta y_R = 0.45$ mm $> \lambda_0/2$. The receivers are sufficiently far from the boundaries of the computational domain to avoid spurious reflections (Figure 1). At each receiver and at each time step, the component of the velocity along x is recorded. Each line corresponds to a particular realization of a random process. Three simulations with different distributions of inclusions provide 135 independent realizations of disorder, ensuring the convergence of the post-processing methods.

At the initial instant, the plane wave lies at $x = 0$ (Figure 1-a). The simulations are stopped after 20000 time steps, when the incident wave has crossed the inclusions. On a Pentium PC at 3 GHz, a time step takes roughly 12 s. Consequently, each simulation takes roughly 66 hours, and it requires 2 Go of RAM.

Post-processing

Coherent field

The discrete coherent field s is obtained by spatially averaging the recorded data through the numerous equivalent realizations of disorder; since ISA considers heterogeneities as discrete scatterers, only receivers located in water are taken into account. The host medium is water, hence the coherent field is acoustic. The coherent field, expressed in the Fourier domain $s(\omega, d_i)$, corresponds to the propagation of a wave in an equivalent homogeneous medium at $N = 42$ regular offsets denoted by $d_i = i\,\Delta x_R$, with $i = 0, ..., N-1$. Phase velocity and attenuation are now extracted from s.

Phase velocity

The phase velocity $c(\omega)$ is computed using the $p-\omega$ transform. This one differs from the spatial Fourier transform by using the slowness $p_0(\omega) = 1/c(\omega)$ as a parameter, which reduces signal-processing errors for the evaluation of phase velocity [3]. The coherent field $s(\omega, d_i)$ is

$$s(\omega, d_i) \;=\; A(\omega, d_i)\mathrm{e}^{-i\omega p_0(\omega) d_i}, \qquad (1)$$

where $A(\omega, d_i)$ is the amplitude spectrum at d_i. The discrete $p-\omega$ stack $\hat{s}(\omega, p)$ is

$$\hat{s}(\omega, p) = \sum_{i=1}^{N} A(\omega, d_i) e^{i\omega(p-p_0(\omega))d_i}. \qquad (2)$$

The computation of $\hat{s}(\omega, p)$ is performed for several values of p. Given ω, the maximum of $|\hat{s}(\omega, p)|$ is reached for $p = p_0(\omega)$, leading to $c(\omega)$.

Damping factor

In the frequency domain, the amplitude spectrum in (1)-(2) can be written as $A(\omega, d_i) \;=\; A_0(\omega)\mathrm{e}^{-\alpha(\omega) d_i}$, where $A_0(\omega)$ is the amplitude of s at the first receiver. The damping parameter $\alpha(\omega)$ is determined by the slope of a least-square linear fit of $\ln(A(\omega, d_i))$. For an incident plane wave, no geometric damping has to be considered.

Results

The comparison between the numerical results and ISA are presented for a density of scatterers of 0.12 rods/mm^2 (surface concentration $6\,\%$), as shown in Figure 1.

Figure 2 presents the ratio $c(\omega)/c_0$, where c_0 is the celerity of waves in water. The theoretical numerical dispersion in water, computed for a 1D mesh, is also plotted. For frequencies lower than 3 MHz, the theoretical phase velocity within ISA and the numerical results are in good agreement. The differences in the higher frequencies may be due to the numerical dispersion. On the whole frequency range, $c(\omega)$ is very close to c_0 [2]. The results for the damping are plotted in figure 3. They are in good agreement, and the differences do not seem to increase with the frequency. The theoretical numerical damping is negligible.

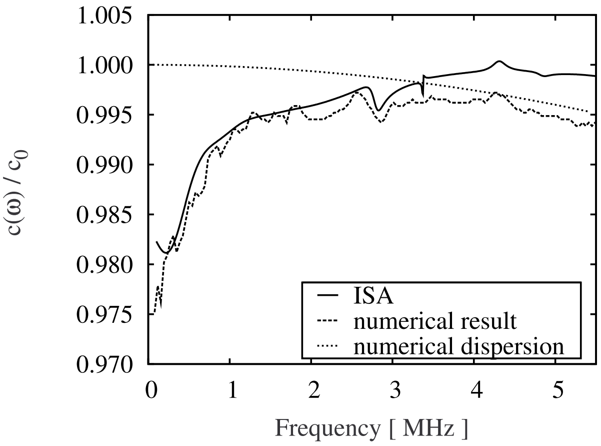

Figure 2: Results for the phase velocity

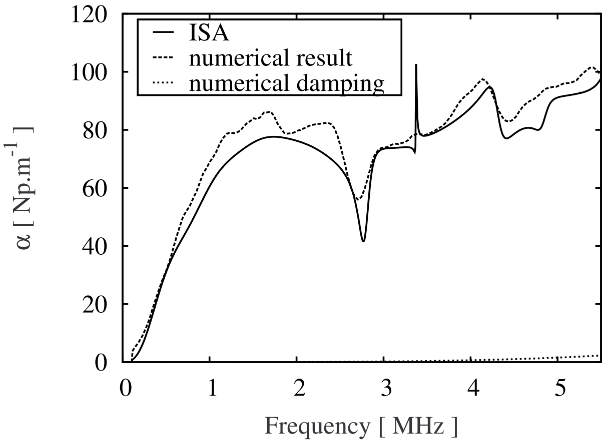

Figure 3: Results for the damping factor

Conclusion and perspectives

The preliminary results proposed here have shown that a numerical methodology allows to recover quite precisely the theoretical behavior predicted by the ISA. Up to now, we cannot clearly state about the small differences observed between both approaches. Are they induced by the approximations underlying the ISA, are they due to an error of protocol, or lastly do they follow from the numerical discretization ? To eliminate with certainty the last hypothesis, we foresee to use more sophisticated schemes (such as fourth-order ADER), and also to compute the solution on a finer grid. For reasons of computational memory, such a convergence study requires the parallelization of the algorithms, which is currently in progress.

An application of the methodology proposed here concerns the concrete. It is a very heterogeneous medium, and it can be considered as aggregates embedded in a cement paste matrix (mortar). Multiple scattering is important, and it is possible to extract a coherent and an incoherent part from measurements [5]. Before using ISA for non-destructive evaluation of concrete, one has to determine whether the hypotheses underlying ISA are satisfied: since aggregates represent 50 % in volume, the medium cannot be considered as dilute. However, the impedance contrast between the aggregates and the mortar is very low. The numerical tools provides a mean to decide whether ISA is still valid or not in that case.

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